Does time-variation matter in the stochastic volatility components for G7 stock returns

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Abstract
This study empirically tests for time variation in the stochastic volatility (SV) components for the G7 stock returns. The time variation in both trend and transitory components of the SV is tested separately and jointly using the unobserved component model and following the approach developed by Chan (2018). Consequently, the computed Bayes factor obtained from the Savage-Dickey density ratio, which circumvents the computation of marginal likelihood, is used to adjudge the performance of each restricted time varying stochastic volatility model without the trend and transitory components against the unrestricted model that allows for same. The empirical evidence supports time variation in the transitory component of SV while the trend component is found to be relatively constant over time. These empirical estimates are not sensitive to data frequency.

Key words: Bayesian; Bayes factor; Transitory component; Trend component; Unobserved Component Model;
JEL Classification: C11; C32; C53; E37; G17
1. Motivation

High risk and uncertainty are inherent characteristics of stock markets particularly those in the G7 countries. Thus, the need to account for volatility when modeling stock returns becomes inevitable. Studies in the extant literature have however, been restricted to the deterministic GARCH-type volatility models (see Lin, 2017; Salisu et al., 2016; Salisu and Ndako, 2017; Kim and Won, 2018; Narayan and Liu, 2018; among others). Following from the exposition of Chan and Grant, 2016 study, a time varying stochastic volatility model, which incorporates time varying parameters to capture the stochastic behaviour of return volatility, is tipped to provide better forecasts. This therefore forms the basis for empirically testing for time-varying stochastic volatility in the stock returns of the G7 countries.

Thus, this study provides answer to the question, “does accounting for time-varying stochastic volatility (examined for both trend and transitory components) matter in predicting stock returns?” This consideration accounts for time variation in model coefficients, which is often neglected owing to the computational burden of pre-testing for same in stochastic volatility with multiple state-space models, of extant volatility studies. By way of circumventing the non-trivial computation of the marginal likelihoods for non-linear state space models with multiple states, Chan (2018) developed an easily implementable technique for testing the time variability of coefficients and volatilities. One of the attractions to this technique is that it employs the Savage-Dickey density ratio, which only requires the estimation of the unrestricted model to compute the Bayes factor (especially useful for nested models) and does not require explicit computation of the marginal likelihoods.

From investment and policy perspectives, developing a modeling framework that better reflects the inherent features of stock returns is important for well-informed investment and policy decisions. More importantly, the effectiveness of macroeconomic policies prominently relies on how policy makers are able to forecast macroeconomic series accurately including stock returns, and therefore the need for refinement of predictive models for forecasting will remain unabated.
The structure of the paper is as follows: Following the introductory section that focused on the motivation for the study, Section 2 presents a brief description of the model set-up with definitions of relevant parameters. The analytical results are presented and discussed in Section 3, while Section 4 concludes the paper.

2. Model set-up

We specify a non-centered parameterization variant of the unobserved component model, drawing from Chan (2018) specification; with the series decomposed into transitory and trend components, such that each component is an independently stochastic volatility process (see Stock and Watson, 2007; Chan, 2018); in equations (1) to (4).

\[ r_t = \tau_0 + \omega_r \tilde{\tau}_t + \epsilon^{1/2}_t (h_0 + \omega_h \tilde{h}_t) \epsilon'_t \]  
\[ \tilde{\tau}_t = \tilde{\tau}_{t-1} + \epsilon^{1/2}_t (g_0 + \omega_g \tilde{g}_t) \epsilon'_t \]  
\[ \tilde{h}_t = \tilde{h}_{t-1} + \epsilon^h_t \]  
\[ \tilde{g}_t = \tilde{g}_{t-1} + \epsilon^g_t \]

where \( r_t \) is the log return of stock price \( \text{asi}_t \), obtained using \( \log(\text{asi}_t/\text{asi}_{t-1}) \); \( \tilde{\tau}_t \), defined as \( \tilde{\tau}_t = (\tau_t - \tau_0)/\omega_r \), is the time-varying intercept, \( \tilde{g}_t \), defined as \( \tilde{g}_t = (g_t - g_0)/\omega_g \), and \( \tilde{h}_t \) defined as \( \tilde{h}_t = (h_t - h_0)/\omega_h \), are time-varying stochastic volatilities in the trend and transitory components, respectively; \( \epsilon'_t \), \( \epsilon^r_t \), \( \epsilon^h_t \) and \( \epsilon^g_t \) are independent Gaussian processes; while \( \tau_0, h_0, g_0, \omega_r, \omega_g \) and \( \omega_h \) are the parameters to be estimated. The respective definitions for \( \tilde{\tau}_t, \tilde{g}_t \), and \( \tilde{h}_t \) follow from Fruhwirth-Schnatter and Wagner (2010), where the corresponding state equations are initialized with \( \tilde{\tau}_1 \sim N(0, V_{\tau} \exp(g_0 + \omega_g \tilde{g}_1)) \), \( \tilde{g}_1 \sim N(0, V_g) \) and \( \tilde{h}_1 \sim N(0, V_h) \), respectively. The \( V_{\tau} \), \( V_g \) and \( V_h \) are the variances that are assumed to be known constants, fixed at 10 (similar to the value used in Stock and Watson, 2007; and Chan, 2018) to enhance empirical applicability and comparison. As consistent with Bayesian estimation, normal priors are assumed for \( \omega_r, \omega_g \) and

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1 This approach has been shown to overcome the challenges associated with the indirect computation of the Bayes factor from the marginal likelihood.
\[ \omega_h \text{ such that } \omega_r \sim N\left(0, V_{\omega_r}\right), \omega_s \sim N\left(0, V_{\omega_s}\right) \text{ and } \omega_h \sim N\left(0, V_{\omega_h}\right) \text{ and } V_{\omega_s} = V_{\omega_s} = V_{\omega_h} = 0.2 \text{ as in Chan (2018).} \]

Following from the main focus of this study, two nested models (restricted and unrestricted) are considered. The restricted model is characterized by the assumption of constant variances with \( \omega_h^2 = \omega_s^2 = 0 \), while the unrestricted model is characterized by variances that are time-varying i.e., \( \omega_h^2 \neq 0 \) and \( \omega_s^2 \neq 0 \) (see specification in equations (1) – (4)). For the purpose of identification and compact representation in the Bayes factor formula, we define the unrestricted and restricted models as \( M_1 \) and \( M_2 \), respectively. We employ the concept of Bayes factor, a prominently used comparative tool when models are nested and more recently, models with time-varying parameters. It is mathematically expressed as \( BF_{12} = \frac{p(r|M_1)}{p(r|M_2)} \), where \( p(r|M_1) \) and \( p(r|M_2) \) are the marginal likelihoods of the first and second models, respectively, given the observed data, in this case stock returns \( r \). Three plausible outcomes are expected, which include \( BF_{12} = 1, BF_{12} < 1 \) or \( BF_{12} > 1 \). The implication of the first is that both unrestricted and restricted models are equally likely, while the second and third stance show preference (higher plausibility of representativeness of the observed data) for the model at the denominator \( (M_2) \) and numerator \( (M_1) \), respectively. Chan (2018) approach\(^2\) for Bayes factor computation, which adopts the Savage-Dickey density ratio (Verdinelli and Wasserman, 1995), is employed in this study and thus specified as:

\(^2\) Considered to be computationally less difficult to implement, since it only requires the estimation of the unrestricted models without explicit computation of the marginal likelihoods. For other marginal likelihood estimation for Gaussian and Non-Gaussian state space models, see Frühwirth-Schnatter (1995); Chan and Eisenstat (2015); Frühwirth-Schnatter and Wagner (2008); Chan and Grant (2016) and Kastner (2015).

\[ BF_{uh} = \frac{p(\omega_h = 0)}{p(\omega_h = 0| r)} \]  
\[ BF_{ug} = \frac{p(\omega_g = 0)}{p(\omega_g = 0| r)} \]  
\[ BF_{uh, g} = \frac{p(\omega_h = \omega_g = 0)}{p(\omega_h = \omega_g = 0| r)} \]

where the subscripts \( u, h \) and \( g \) are used to indicate the models being compared with the Bayes factors in equation (5) to equation (7), such that \( u \) represents the unrestricted model \( (\omega_h \neq 0) \); \( h \) represents the restricted model with fixed transitory component \( (\omega_h = 0) \), \( g \) represents the restricted model with fixed trend component \( (\omega_g = 0) \); and the combined subscripts, \( hg \), denotes the case where both the transitory and trend components are simultaneous restricted to zero \( (\omega_h = \omega_g = 0) \). Consequently, \( BF_{uh} \) corresponds to a comparison of the unrestricted and restricted (fixed transitory component), \( BF_{ug} \) corresponds to a comparison of the unrestricted and restricted (fixed trend component) and \( BF_{uh, g} \) corresponds to a comparison of the unrestricted and restricted (simultaneously restricted transitory and trend component).

3. Results And Discussion

We consider the stock returns of the G7 countries (Canada, France, Germany, Italy, Japan, UK and USA) obtained from the Bloomberg terminal for a time period spanning 1998 to 2018. The series were collected on three different frequencies (daily, weekly and monthly) for analytical and robustness purposes. We proceed with the estimation of the afore-described unobserved component model for the restricted and unrestricted (with individually and simultaneously fixed transitory and trend components) models, and consequently obtain the corresponding log of Bayes factor (see results in Table 1).

Consistent with the conventional Bayes factor, values greater than unity are indicative of the likelihood of the model in the numerator being well suited for the observed data, while the reverse indicates preference for the denominator model. In this study, our numerator and denominator
models are, respectively, the unrestricted and restricted models of different constructs as earlier defined. On the log Bayes factor criteria, the unrestricted model would be preferred whenever the log Bayes factor is greater than zero, otherwise, the restricted model being compared would be preferred. The outcome is observed across different data frequencies ranging from daily, weekly to monthly.

Table 1: Log Bayes Factor Output of the G7 Countries’ Return Series

<table>
<thead>
<tr>
<th>Country</th>
<th>Daily</th>
<th>Weekly</th>
<th>Monthly</th>
</tr>
</thead>
<tbody>
<tr>
<td>CANADA</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\log BF_{uh}$</td>
<td>265.9 (7.90)</td>
<td>310.3 (6.62)</td>
<td>17.9 (3.02)</td>
</tr>
<tr>
<td>$\log BF_{ug}$</td>
<td>-3.6 (0.39)</td>
<td>-3.8 (0.06)</td>
<td>-1.9 (0.11)</td>
</tr>
<tr>
<td>$\log BF_{a,gb}$</td>
<td>280.8 (7.80)</td>
<td>311.4 (5.89)</td>
<td>21.6 (3.51)</td>
</tr>
<tr>
<td>FRANCE</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\log BF_{uh}$</td>
<td>324.1 (5.98)</td>
<td>267.9 (6.82)</td>
<td>33.1 (3.76)</td>
</tr>
<tr>
<td>$\log BF_{ug}$</td>
<td>-2.5 (0.41)</td>
<td>-3.8 (0.07)</td>
<td>-1.9 (0.14)</td>
</tr>
<tr>
<td>$\log BF_{a,gb}$</td>
<td>343.7 (7.86)</td>
<td>269.0 (5.93)</td>
<td>39.3 (4.43)</td>
</tr>
<tr>
<td>GERMANY</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\log BF_{uh}$</td>
<td>298.9 (11.43)</td>
<td>246.4 (7.88)</td>
<td>48.3 (3.50)</td>
</tr>
<tr>
<td>$\log BF_{ug}$</td>
<td>-3.0 (0.36)</td>
<td>-3.7 (0.09)</td>
<td>-2.0 (0.08)</td>
</tr>
<tr>
<td>$\log BF_{a,gb}$</td>
<td>318.1 (10.44)</td>
<td>249.7 (7.29)</td>
<td>54.5 (4.71)</td>
</tr>
<tr>
<td>ITALY</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\log BF_{uh}$</td>
<td>278.3 (6.74)</td>
<td>276.3 (5.73)</td>
<td>36.3 (2.83)</td>
</tr>
<tr>
<td>$\log BF_{ug}$</td>
<td>2.0 (1.04)</td>
<td>-3.0 (0.11)</td>
<td>-2.0 (0.11)</td>
</tr>
<tr>
<td>$\log BF_{a,gb}$</td>
<td>315.1 (6.08)</td>
<td>289.5 (6.59)</td>
<td>40.6 (2.75)</td>
</tr>
<tr>
<td>JAPAN</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\log BF_{uh}$</td>
<td>103.2 (3.16)</td>
<td>64.2 (3.95)</td>
<td>15.4 (3.13)</td>
</tr>
<tr>
<td>$\log BF_{ug}$</td>
<td>-4.9 (0.08)</td>
<td>-3.6 (0.08)</td>
<td>-2.1 (0.08)</td>
</tr>
<tr>
<td>$\log BF_{a,gb}$</td>
<td>108.4 (3.41)</td>
<td>63.4 (3.56)</td>
<td>18.7 (3.19)</td>
</tr>
<tr>
<td>UK</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\log BF_{uh}$</td>
<td>0.2 (0.94)</td>
<td>155.6 (7.90)</td>
<td>41.2 (4.34)</td>
</tr>
<tr>
<td>$\log BF_{ug}$</td>
<td>-5.1 (0.07)</td>
<td>-3.8 (0.08)</td>
<td>-0.9 (0.17)</td>
</tr>
<tr>
<td>$\log BF_{a,gb}$</td>
<td>-4.3 (1.21)</td>
<td>155.2 (8.27)</td>
<td>62.3 (5.73)</td>
</tr>
<tr>
<td>USA</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\log BF_{uh}$</td>
<td>219.0 (5.43)</td>
<td>153.9 (4.84)</td>
<td>23.2 (2.93)</td>
</tr>
<tr>
<td>$\log BF_{ug}$</td>
<td>-4.1 (0.13)</td>
<td>-3.8 (0.04)</td>
<td>-2.2 (0.09)</td>
</tr>
<tr>
<td>$\log BF_{a,gb}$</td>
<td>227.0 (5.09)</td>
<td>155.4 (5.11)</td>
<td>28.1 (5.25)</td>
</tr>
</tbody>
</table>

Note: The figures in each cell represent the log of the Bayes factor and their corresponding numerical standard errors in parentheses (required to gauge accuracy of the estimates). The Bayes factor compares the restricted model with an unrestricted model, such that $BF > 1 \text{ or } \log BF > 0$ indicates preference in favour of the unrestricted model (having stochastic volatility process in at least one component (transitory and/or trend)). The subscript for each Bayes factor represents the models being compared.

On the log Bayes factor - $\log BF_{uh}$ which involves comparing the unrestricted and the restricted models with fixed transitory component, we find positive (large) values across data frequencies
and across the G7 countries. In other words, these results are not sensitive to data frequency or stock return series used, as similar stances are observed across the data frequencies and G7 countries. This implies consistent preference in favour of the unrestricted model over the restricted model with time-invariant transitory component. By implication, allowing for time-varying parameter in the transitory component of the stochastic volatility process will improve the forecast performance of the predictive model for stock returns.

However, the reverse is the case for the trend component as the log Bayes factors, $\log BF_{ug}$, obtained under this comparison are negative and relatively small for all the considered data frequencies and G7 countries’ stock return series examined except for Italy, when daily frequency is used. Thus, the restricted model with fixed trend component is however, favoured over the unrestricted model and by implication, restricting the stochastic volatility process in the trend component might, although weakly, be more suited to the stock return series for the G7 countries than its unrestricted counterpart.

On the third scenario with the log Bayes factor - $\log BF_{w,gh}$ which both the transitory and trend components in the same framework, we find similar results as in the earlier case, where the restricted model with fixed transitory component is compared with the unrestricted model. The obtained log Bayes factors, $\log BF_{w,gh}$, are positive for all G7 countries except UK when the daily frequency is used (where it is negative but relatively weak). We find here again a strong evidence that supports the consideration of stochastic volatility process in at least one component, whether transitory and/or trend, when predicting stock returns of the G7 countries, regardless of the data frequency considered.

We further display diagrammatically, the behaviour of the standard deviations of the transitory and trend components for the G7 countries over the period under consideration (see Figure 1). Different levels of stability could be vividly observed between the transitory and trend components, such that the latter seems to be more stable (especially, with respect to the monthly frequency) while the former exhibits more variations over time. This further reinforces the need to
account for the inherent time-variation in the stochastic volatility process with transitory component when dealing with stock returns.

<table>
<thead>
<tr>
<th>G7</th>
<th>Daily $\exp(h/2)$</th>
<th>$\exp(g/2)$</th>
<th>Weekly $\exp(h/2)$</th>
<th>$\exp(g/2)$</th>
<th>Monthly $\exp(h/2)$</th>
<th>$\exp(g/2)$</th>
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<td>CANADA</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
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<tr>
<td>FRANCE</td>
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<td><img src="image41" alt="Graph" /></td>
<td><img src="image42" alt="Graph" /></td>
</tr>
</tbody>
</table>

Figure 1: Frequencies of Time-Varying Standard Deviation of Transitory $\exp(h/2)$ and Trend $\exp(g/2)$ Components for Stock Returns of G7 Countries.  
Note: The shaded region represent 90% credible interval.
4. Conclusion

This study tests for time-varying stochastic volatility in stock returns of the G7 countries and follows the Chan (2018) approach, which adopts a computationally flexible Savage-Dickey density ratio, to compute the relevant Bayes factor. Summarily, our results reveal substantial evidence in support of time-varying stochastic volatility process with transitory component. With results that are not sensitive to data frequency, we conclusively state that models for stock returns that incorporate time-varying stochastic volatility would yield better forecast results than models that ignore same.

References
Salisu, A. A. and Ndako, U. B. (2017). Forecasting the return volatility of European equity markets under different market conditions: A GARCH-MIDAS approach- Centre for Econometric and Allied Research, University of Ibadan Working Papers Series, CWPS 0028