MODELLING AND FORECASTING VOLATILITY

(SYMMETRIC GARCH MODELS)

By

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Outline of the Presentation

- ❑ Background
- ❑ Symmetric GARCH Models:
  - ➢ ARCH Model
  - ➢ GARCH Model
- ❑ Model Selection Criteria
- ❑ Forecasting with Volatility Models
- ❑ Empirical Applications

Background

- Financial time series such as stock returns, exchange rate, inflation, etc. are available at a high frequency, hence, exhibit random walk and time varying volatility (heteroscedasticity).

- Reasons for modeling and forecasting volatility.
- Stock Market Investors: Huge Losses or Gains due to volatility of stock prices. Need to analyze the risk of holding an asset or the value of an option.

Background contd.

- International Trade (Importers, Exporters and FOREX Traders): Huge losses or profits due to volatility of exchange rate.
- How do we model financial time series with evidence of volatility?

Types of Univariate Volatility Models

- Autoregressive Conditional Heteroscedasticity (ARCH) Model
- Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Model.
- The ARCH & GARCH Models are referred to as Symmetric GARCH Models. Why?
- Further Extensions: Asymmetric GARCH Models; Integrated & Fractionally Integrated GARCH Models and Multivariate GARCH Models.
- However, the main objective of this lecture is to cover the Symmetric Volatility models (ARCH and GARCH Models).

Autoregressive Conditional Heteroscedasticity (ARCH) Model

- The ARCH Model was developed by Engle (1982)
- Autoregressive implies that the series depends on its past values.
- Volatility of time series in econometrics is referred to as conditional variance.
- Time varying volatility is referred to as conditional heteroscedasticity.
- These two features combine to give **ARCH Model.**

ARCH Model contd.

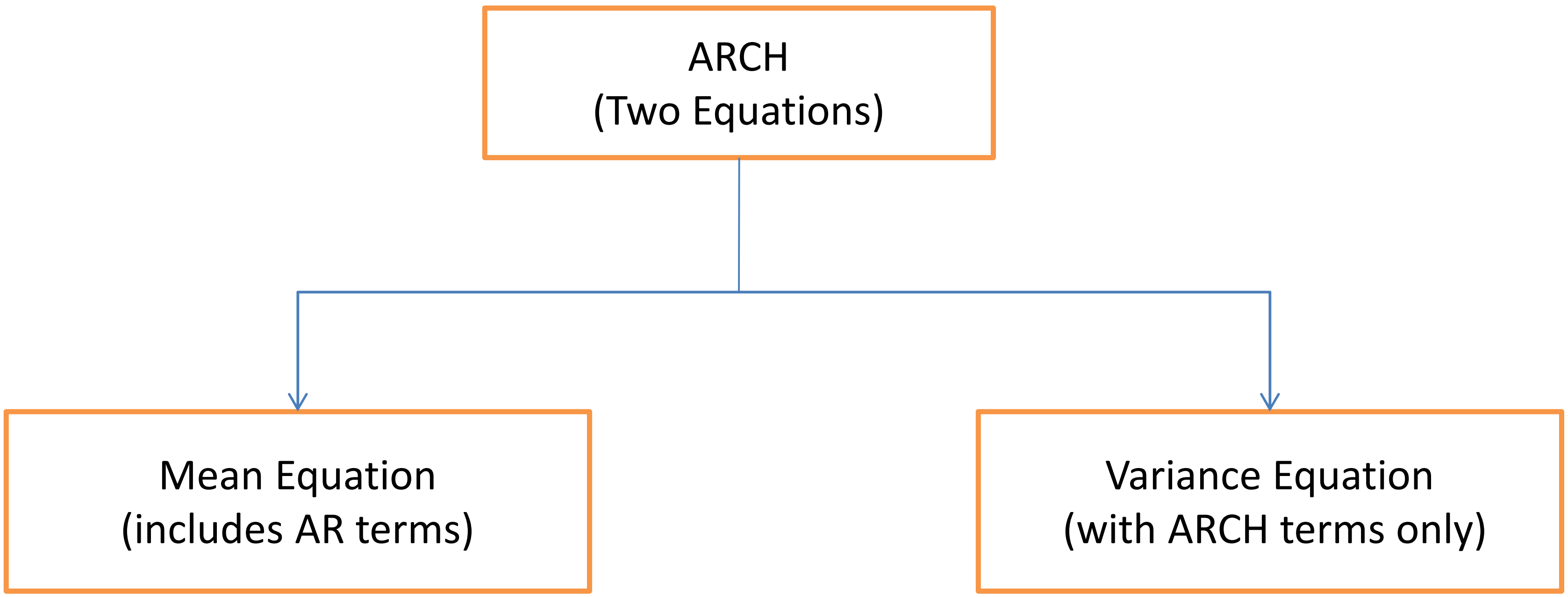
❑ Also note:

- Autoregressive describes a feedback mechanism that incorporates past observations into the present.

- Conditional implies dependence on the observations of the immediate past;

ARCH Model contd.

Also important to note:



ARCH Model contd.

- Thus, ARCH is a mechanism that includes past variances in the explanation of future variances.
- ARCH is a time-series technique that allows researchers to model the serial dependence of volatility.

ARCH Model contd.

- Estimation Procedure for ARCH (Volatility) Modelling:

- Testing for ARCH effects: Is the series in question volatile?
- Estimation with ARCH-type Models: This is considered only if the series is volatile.
- Post Estimation test: This is carried out to verify if the estimated ARCH-type model has captured the ARCH effects in the series. It involves testing for ARCH effects after estimation.

ARCH Model contd.

❑ Testing for ARCH Effects

➢ Estimate the Mean Equation (AR) model by OLS:

✓ The simplest version is AR(1):

$$z_t = \alpha + \varphi z_{t-1} + \upsilon_t; \quad \upsilon_t \sim \text{IID}\left(0, \sigma^2\right)$$

✓ Higher order AR models $\left[\text{AR}(p)\right]$ can be represented as:

$$z_t = \alpha + \sum_{i=1}^{p} \varphi_i z_{t-i} + \upsilon_t; \quad \upsilon_t \sim \text{IID}\left(0, \sigma^2\right)$$

ARCH Model contd.

- Obtain the fitted residuals of the regression model:

$$\hat{\upsilon}_t = z_t - \hat{\alpha} - \hat{\varphi} z_{t-1} \text{ for AR(1)}$$

$$\hat{\upsilon}_t = z_t - \hat{\alpha} - \sum_{i=1}^{p} \hat{\varphi}_i z_{t-i} \text{ for AR}(p)$$

$$\hat{\upsilon}_t \Rightarrow \text{ Fitted Residuals}$$

ARCH Model contd.

- Regress the square of the fitted residuals on a constant and lags of the squared residuals:

$$\hat{\upsilon}_t^2 = \alpha + \lambda \hat{\upsilon}_{t-1}^2 \text{ for ARCH}(1) \text{effect}$$

$$\hat{\upsilon}_t^2 = \alpha + \sum_{i=1}^{p} \lambda_i \hat{\upsilon}_{t-i}^2 \text{ for ARCH}(p) \text{effects}$$

$$\text{H}_0\text{: } \lambda = 0 \text{ for ARCH}(1) \Rightarrow \text{ No ARCH}(1) \text{ effects}$$

$$\text{H}_0\text{: } \lambda_1 = \lambda_2 = \cdots = \lambda_p = 0 \text{ for ARCH}(p)$$

$$\Rightarrow \text{ No ARCH}(p) \text{ effects}$$

ARCH Model contd.

- Thus, if ARCH effects are present, the estimated parameters should be statistically
  different from zero
  $\Rightarrow$ the series is volatile
- However, if ARCH effects are not present, then, the estimated parameters should be statistically insignificant from zero
  $\Rightarrow$ the series is not volatile

ARCH Model contd.

- The test statistics for the null hypothesis:

$$F \text{ - test } \left[\text{df} = \left(p\text{-}1, \text{N-}p \right) \right];$$

$$\text{n}R^2 \sim x_p^2 \ \left(\text{df} = p \right)$$

- The null hypothesis of no ARCH effects is rejected if the probability values (p-values) of these tests are less than any of the conventional levels of statistical significance (10%, 5%, and 1%).

ARCH Model contd.

- By implication, the null hypothesis of no ARCH effects is not rejected if the probability values (pv) of these tests are greater than any of the conventional levels of statistical significance (10%, 5%, and 1%).
- Thus:

$$\text{if pv} < 0.10;\ 0.05;\ \text{or } 0.01$$

$$\text{Decision: Reject } H_0 \Rightarrow \text{Series is volatile}$$

ARCH Model contd.

- However,

  if pv $>$ 0.10; 0.05; or 0.01

  Decision: Do not Reject H_0

  $\Rightarrow$ Series is not volatile

ARCH Model contd.

- Note the following:
  - A precondition for volatility modelling is the test for ARCH effects;
  - In other words, volatility models can only be used when the series has been empirically tested to be volatile.
  - The procedure for testing for ARCH effects as earlier described is consistent with the ARCH LM test proposed by Engle (1982);
  - Thus, when testing for ARCH effects using any of the standard econometric softwares, the ARCH LM test should be employed.

ARCH Model contd.

❑ ARCH Model Estimation

- Assuming that the series is volatile based on the ARCH LM test, how can this series be modelled?
- Two equations are involved:
- The Mean Equation:

$$\text{AR}(1) \;\rightarrow z_t = \alpha + \varphi z_{t-1} + \upsilon_t;$$

$$\text{AR}(p) \rightarrow z_t = \alpha + \sum_{i=1}^{p} \varphi_i z_{t-i} + \upsilon_t;$$

ARCH Model contd.

- The Variance Equation:

$$\text{ARCH}(1) \rightarrow \sigma_t^2 = \lambda_0 + \lambda_1 \upsilon_{t-1}^2$$

$$\text{ARCH}(p) \rightarrow \sigma_t^2 = \lambda_0 + \sum_{i=1}^{p} \lambda_i \upsilon_{t-i}^2$$

$$0 \leq \sum_{i=1}^{p} \lambda_i < 1 \Rightarrow \text{Stationary Series}$$

ARCH Model contd.

$$\upsilon_{t-i}^{2} \Rightarrow \text{ ARCH terms}$$

$$\sum_{i=1}^{p} \lambda_i \to 1 \Rightarrow \text{ Slow Mean Reversion}$$

$$\sum_{i=1}^{p} \lambda_i \to 0 \Rightarrow \text{ Fast Mean Reversion}$$

ARCH Model contd.

- How can we determine the optimal lag for the ARCH(p)-type Models?
- There are three standard criteria:
- Akaike Information Criterion (AIC)

$$\text{AIC}\left(g\right) = \text{Log}\left(\hat{\varepsilon}'\hat{\varepsilon}/n\right) + 2g/n$$

- Schwartz Information Criterion (SIC)

$$\text{SIC}\left(g\right) = \text{Log}\left(\hat{\varepsilon}'\hat{\varepsilon}/n\right) + g\log n/n$$

ARCH Model contd.

- Hannan-Quinn Information Criterion (HQC)

$$\text{HQC}(g) = \text{Log}\left(\hat{\varepsilon}'\hat{\varepsilon}/n\right) + 2g \log\log n/n$$

- Decision: The lag that produces the least AIC/SIC/HQC is assumed to be the optimal lag.
- As a consequence, the model with the optimal lag is assumed to be the best model for the ARCH(p)-type models.
- These criteria can also be used to determine the best error distribution for the series.

ARCH Model contd.

- **Post Estimation Test:**
  - This is required to validate the choice of ARCH-type models when confronted with volatility.
  - This test follows the procedure for testing ARCH effects.
  - The only difference however is the fact that the test is carried out after estimation of ARCH-type Models.

- Now, let us consider empirical applications of the ARCH(*p*)-type models using Eviews

GARCH Model

- The GARCH model was originally proposed by Bollerslev (1986).
- The Model is an extension of Engle (1982) ARCH Model.
- The model also has two equations: Mean Equation and Variance Equation.
- The Mean Equations in ARCH and GARCH are the same.
- However, the Conditional variance equations differ.

GARCH Model contd.

- The conditional variance equation incorporates lags of the conditional variance as regressors in the conditional variance equation in addition to lags of the squared error terms.
- Thus, for GARCH Models:

  Cond. variance = f(ARCH terms, GARCH terms)
- Recall, for ARCH-type Models:
- Cond. variance = f(ARCH terms only)

GARCH Model contd.

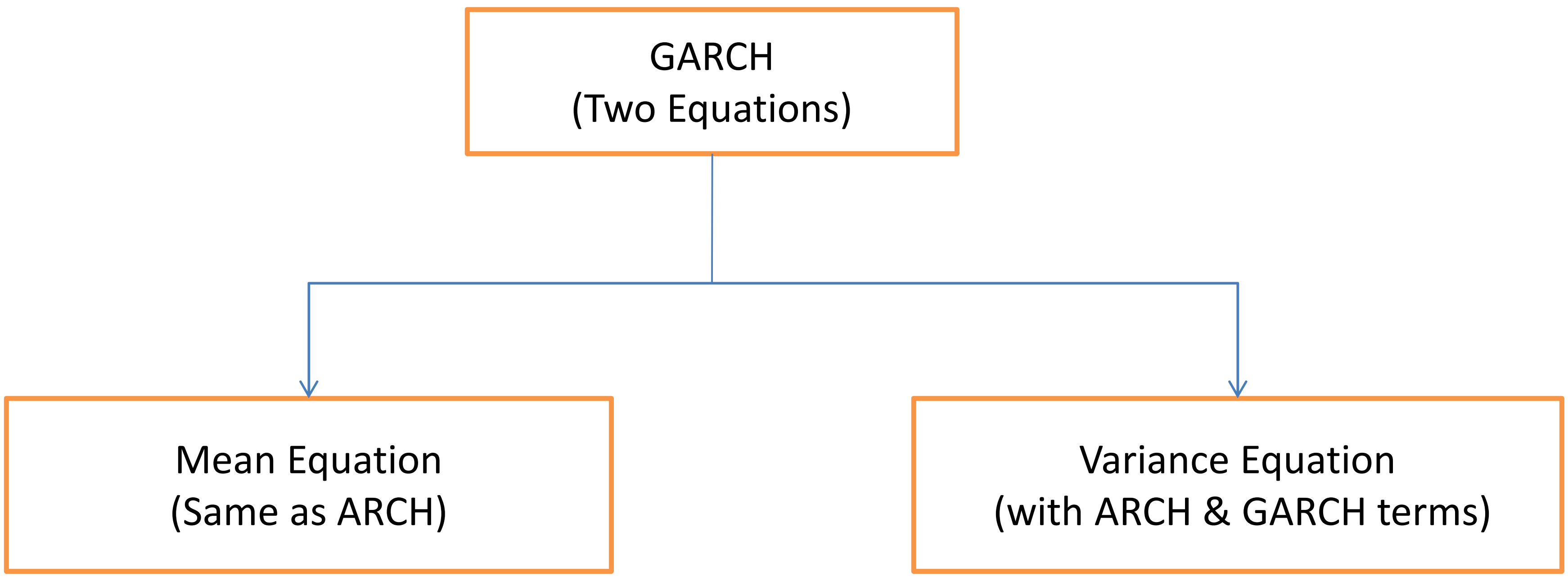
- The simplest version of GARCH model is the GARCH (1,1) model:

$$\sigma_t^2 = \lambda_0 + \lambda_1 \upsilon_{t-1}^2 + \gamma_1 \sigma_{t-1}^2$$

- The (1,1) in parenthesis is a standard notation for the ARCH and GARCH terms.
- The first number (1) refers to first order ARCH term or first order AR term,
- while the second number (1) refers to first order GARCH term or first moving average term [MA(1)].

GARCH Model contd.

GARCH Modelling Framework:



GARCH Model contd.

GARCH(p,q) Model:

$$\sigma_t^2 = \lambda_0 + \lambda_1\upsilon_{t-1}^2 + \lambda_2\upsilon_{t-2}^2 + \cdots + \lambda_p\upsilon_{t-p}^2$$

$$+ \gamma_1\sigma_{t-1}^2 + \gamma_2\sigma_{t-2}^2 + \cdots + \gamma_q\sigma_{t-q}^2$$

$$\sigma_t^2 = \lambda_0 + \sum_{i=1}^{p} \lambda_i \upsilon_{t-i}^2 + \sum_{j=1}^{q} \gamma_j \sigma_{t-j}^2$$

$$0 \le \sum_{i=1}^{p} \lambda_i + \sum_{j=1}^{q} \gamma_j < 1 \Rightarrow \text{Stationary Series}$$

GARCH Model contd.

- On mean reverting process:

$$\sum_{i=1}^{p}\lambda_i + \sum_{j=1}^{q}\gamma_j \rightarrow 1 \Rightarrow \text{ Slow Mean Reversion}$$

$$\sum_{i=1}^{p}\lambda_i + \sum_{j=1}^{q}\gamma_j \rightarrow 0 \Rightarrow \text{ Fast Mean Reversion}$$

GARCH Model contd.

Estimation Procedure:

- Testing for ARCH effects: Same as ARCH-type models
- Estimation: The only difference is in the conditional variance equation.
- Post estimation: Same as ARCH-type models.

Now, let us consider empirical applications of the GARCH model using Eviews

Model Selection Criteria

- When a researcher is confronted with these univariate volatility models, how can the best model be determined?
- The earlier explained standard criteria can be used. That is:
- The Akaike Information Criterion (AIC)
- Schwartz Information Criterion (SIC)
- Hannan-Quinn Information Criterion (HQC)

Model Selection Criteria contd.

- Among these criteria, the SIC is often the preferred method as it gives the heaviest penalties for loss of degrees of freedom. Thus, model with the least value of SIC is assumed to give the best fit among the competing models.

Forecasting Volatility Models

- Essentially, the forecast allows the projection of s-step ahead of T (the sample size) for the series. The forecast function for s-steps ahead can be expressed as:

$$E\left(z_{T+s}|\Omega_T\right)=\hat{\eta}+\hat{\delta}_1 z_{T+s-1}; \quad \varepsilon_t \sim \mathrm{IID}\left(0,\sigma^2\right)$$

- One-step ahead forecast (i.e. s=1):

$$E\left(z_{T+1}|\Omega_T\right)=\hat{\eta}+\hat{\delta}_1 z_T$$

- Two-step ahead forecast (i.e. s=2):

$$E\left(z_{T+2}|\Omega_T\right)=\hat{\eta}+\hat{\delta}_1 z_{T+1}$$

Measures of forecast performance

❑Standard measures are:

➢Root Mean Square Error (RMSE)

$$\text{RMSE} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\hat{z}_t - z_t\right)}$$

➢Mean Absolute Error (MAE):

$$\text{MAE} = \frac{1}{T}\sum_{t=1}^{T}\left|\hat{z}_t - z_t\right|$$

➢Absolute Percent Error (MAPE):

$$\text{MAPE} = \frac{1}{T}\sum_{t=1}^{T}\hat{z}_t - z_t / z_t$$

Measures of forecast performance Contd.

- Theil's Inequality Coefficient (TIC)

$$\text{TIC} = \frac{\sqrt{\frac{1}{\text{T}}\sum_{t=1}^{T}\left(\hat{z}_t - z_t\right)^2}}{\sqrt{\frac{1}{\text{T}}\sum_{t=1}^{T}\left(\hat{z}_t\right)^2} - \sqrt{\frac{1}{\text{T}}\sum_{t=1}^{T}\left(z_t\right)^2}}$$

✓ The volatility model **with the least** RMSE, MAE and MAPE and **highest** TIC statistics is the best forecasting model

Thank you