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A new procedure for pre-testing the distribution properties of Stock returns

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Abstract

The study offers a new procedure that helps determine the best distribution prior to modeling stock returns with GARCH-type models. Specifically, it demonstrates that pre-testing the residuals of stock returns for the best distribution can help to identify the appropriate GARCH error distribution regardless of the choice of GARCH-type model. This approach is robust to alternative data frequencies and different stock markets such as those of G7 countries.

Keywords: Stock returns; GARCH-type models; Error distributions

Introduction

Aside risk, one of the most studied topics in the finance literature is the rate of return on investments (bonds, stocks, equity, debentures, among the likes). The aim of every investor is to maximize returns on investments. The literature on stock returns is huge and the present paper does not intend to review them. However, Atilgan et al. (2015) and Reddy and Narayan (2016) provide a detailed literature survey on stock returns. In spite of the level of sophistication of models for stock returns, the increasing dynamics of the global economy with attendant consequences on the stock markets have continued to challenge researchers on how best to reflect these dynamics in the empirical analyses of stock returns.

In this study, further attempts are made to develop an approach that can help pre-determine the distribution that best fits stock returns thus, improving the precision of its predictive model. Meanwhile, earlier literature including stock option valuation method by Black and Scholes (1973) and Capital asset pricing model by Sharpe (1964) models assumed normal distribution for the rate of return in value at risk. Aside for the ease, fast and simplified approach of calculations and or estimations, there is no other justifiable reason to uphold this assumption. Occurrences in the global economies such as demand for safe assets (Bernanke, 2005), global financial crisis, Asian 1990s and European 2000s debt crisis, asymmetry in the global financial markets, global imbalances and saving glut hypothesis (Bernanke, 2005), to mention a few seem to have nullified the assumption of normal distribution of the series. The earlier literature had improved on this by subjecting the series to several non-normal distribution tests (see Consiglio et al, 2002, Czyżycki, 2013; Purczyński and Bednarz-Okrzynska, 2014) and their findings further suggest the need to formalize the procedure for determining the best distribution for stock returns.

Thus, in this study, we offer two novelties to the extant literature: first, we inquire whether pre-testing for the distribution property of the stock returns matters in choosing appropriate distribution for the GARCH-type models¹. This involves two stages: (i) we estimate a typical mean equation for stock returns and subject the residual to different distribution analyses; (ii) We then investigate whether the best distribution for the GARCH-type models picks the best

¹ The GARCH-type models are considered for modeling stock returns here due to the presence of conditional heteroscedasticity effects in the series (see the preliminary analyses in Section 2.0).

distribution obtained from the pre-test. This provides answers to the inquiry in terms of whether pre-testing for the distribution of the series can serve as a precondition for selecting the appropriate distribution when modelling and forecasting stock returns. Second, for the purpose of making reasonable generalizations about our findings, we consider the G7 stock markets and three data frequencies (daily, weekly and monthly). Our findings further justify the need to include the residual distribution test in the list of preliminary tests when modeling and forecasting stock returns.

We structure the remainder of the paper as follows. Methodology and data issues are discussed in the second section. In the third section, we present the results, while the fourth section concludes the paper.

2. Methodology, Data and Preliminary Analysis

2.1 Methodology

The first aspect of the methodology dwelled on the pre-testing for the distribution of stock returns, while the second bit presents GARCH analyses with different models and distributions. For robustness purposes, we use the same set of distribution for both stages.

For the first stage analysis, we estimate the model below:

$$r_t = \alpha + \rho r_{t-1} + \varepsilon_t \quad \text{where} \quad r_t = \log(p_t/p_{t-1}) \quad (1)$$

where p_t is the aggregate stock price and r_t is the corresponding return series. The error term is then fitted into the following distributions: Gaussian; Student-t; Generalised Error; Modified Cauchy; Laplace; Logistic; Hansens Skew-t; Gram-Charlier expansion series with constant higher moments; Rayleigh; and Extreme Value Distribution Type 1.

In the second stage of the analysis, eight GARCH-models were considered whose choice is determined largely by the selected distributions and for convenience, we allow for first lag in the relevant variables both in the mean and variance equations. ARMA (1,1) is the mean equation common to the selected GARCH-type model: $r_t = \alpha + \rho r_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1}$ where $\rho \neq 0$; $\theta \neq 0$ and

$\varepsilon_t = \sigma_t e_t$; but e_t follows different distributions as previously mentioned while σ_t is the conditional standard deviation. The corresponding variance equation of the latter varies for the different GARCH-type models based on the underlying characteristics for ARCH and GARCH terms but with a common lag combination (1,1) respectively. The considered GARCH-type models are: GARCH: $\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$ where $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$; GJR-GARCH: $\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \gamma \varepsilon_{t-1}^2 I_{t-1} + \beta \sigma_{t-1}^2$ where $I_{t-1} = 1$ if $\varepsilon_{t-1} < 0$ and zero(0) if $\varepsilon_{t-1} \geq 0$; EGARCH: $\ln \sigma_t^2 = \omega + \alpha \left| u_{t-1} / \sqrt{\sigma_{t-1}^2} \right| + \gamma \left(u_{t-1} / \sqrt{\sigma_{t-1}^2} \right) + \beta \ln \sigma_{t-1}^2$; NARCH (Nonlinear ARCH): $\sigma_t^\delta = \omega + \alpha |\varepsilon_{t-1}|^\delta + \beta \sigma_{t-1}^\delta$; NGARCH (Nonlinear GARCH): $\sigma_t^\delta = \left(\omega + \alpha |\varepsilon_{t-1}|^\delta + \beta \sigma_{t-1}^\delta \right)^{1/\delta}$; AGARCH (Asymmetric GARCH): $\sigma_t^2 = \omega + \alpha (\varepsilon_{t-1} + \gamma)^2 + \beta \sigma_{t-1}^2$; APGARCH (Asymmetric Power GARCH): $\sigma_t^\delta = \left(\omega + \alpha (|\varepsilon_{t-1}| - \gamma \varepsilon_{t-1})^\delta + \beta \sigma_{t-1}^\delta \right)^{1/\delta}$ and NAGARCH (Nonlinear Asymmetric GARCH): $\sigma_t^2 = \omega + \alpha \left((\varepsilon_{t-1} / \sigma_{t-1}) + \gamma \right)^2 + \beta \sigma_{t-1}^2$ where $-1 < \gamma < 1$.

2.2 Data

The scope of this study is limited to the G7 countries (Canada, France, Germany, Italy, Japan, United States of America (US) and the United Kingdom (UK)). Daily data for the period 01/01/2000 to 16/02/2018 is compiled. Other frequencies, such as weekly and monthly, were also explored. This is to examine the robustness of our results. The availability of data dictated the selected scope. Our data is sourced from Bloomberg terminal.

2.3 Preliminary results

Table 1 presents results of the descriptive statistics. Over all, the average stock return is quite small across the countries under investigation. There are two distinctive features from these statistics: first, the lower the frequency, the higher is the return of the series with the exception of US and UK. Second, only UK has a negative return. Gregoriou et al. (2009) linked this scenario to the tightening of the monetary policy by the Bank of England. Across countries and frequencies, the series: (i) mildly deviate away from their mean, based on the standard deviation statistics; (ii) are negatively skewed and are also leptokurtic.

Table 1: Data Description

Country	Freq.	Mean	Std. Dev	Skew.	Kurtosis	Jaq. Ber.	Q Stat	Q2 Stat	ARCH LM Test
U.S	Daily	0.0002	0.012	-0.025	11.79	14694	134**	98**	4549
	Weekly	0.0001	0.024	-0.569	8.998	1469	45*	26*	933.9
	Monthly	0.004	0.042	-0.583	4.202	25.387	1.786	0.076	210.70
Canada	Daily	0.0003	0.014	-0.352	11.664	14344	86**	46**	4529
	Weekly	0.001	0.031	-0.771	10.321	2206	3.687*	1.787	917
	Monthly	0.005	0.057	-0.553	5.544	69.544	21	8.907	193.07
UK	Daily	-0.0001	0.011	-1.051	11.166	13583	159.56*	5.676**	4555
	Weekly	-0.0003	0.028	-1.265	9.614	1976	4.636*	1.656	900.3
	Monthly	-0.001	0.072	-0.839	6.153	114	16.226*	0.0941	148.7
France	Daily	0.0001	0.016	0.160	9.532	8269	1.235	0.643	4593
	Weekly	0.0001	0.032	-0.562	7.868	984	0.170	0.394	907.1
	Monthly	0.0003	0.061	-0.454	3.712	12.092	1.516	0.001	190.06
Germany	Daily	0.0003	0.072	0.072	7.984	4773	0.471	1.443	4887
	Weekly	0.001	0.034	-0.436	7.324	765	0.534	2.754**	924.4
	Monthly	0.007	0.069	-0.468	4.562	29.519	1.767	0.163	200.76
Italy	Daily	0.0001	0.016	-0.038	9.798	7418	0.754	0.0852	3832
	Weekly	0.0008	0.034	-0.876	7.648	811	2.621	7.312*	771.6
	Monthly	0.004	0.068	-0.407	3.560	7.331	3.845**	0.377	167.14
Japan	Daily	0.0001	0.015	-0.155	6.897	2836	46.455*	8.365	4411
	Weekly	0.0005	0.028	-0.432	6.395	483	0.197	2.480	909.9
	Monthly	0.0009	0.053	-0.791	4.163	34	4.565**	1.986	184.00

Source: Authors' Computation.

Note: The reported values for the serial correlation are the Ljung-Box Q-statistics and ARCH-LM test F-statistics for the conditional heteroscedasticity test. We consider three different lag lengths (k) of 2, 5 and 10 for robustness purpose. The null hypothesis for the autocorrelation test is that there is no serial correlation, while the null for the ARCH-LM test is that there is no conditional heteroscedasticity. *** indicates significance at 1%.

3. Results

A two-step procedure is required to achieve the objective of the study. In the first step, a typical mean equation is estimated without accounting for the GARCH error. This is the pre-testing of the distribution of the series. Consequently, we test for the appropriate distribution of the residuals. The next stage is to examine the preferred distribution in the former case will still remain dominant for the GARCH analyses irrespective of the choice of GARCH-type model. For both stages, statistics of the log-likelihood (LL) and Akaike Information Criteria (AIC) are presented. The best distribution in both cases is selected based on the one with the least AIC values and the highest LL statistics.

For the first stage, Generalized Error Distribution (GED) is the most preferred in describing stock return for all the countries across different data frequencies. Table 2 presents these results. For the second stage and to allow for consistency, we examined eight different types of GARCH-type models for the same set of distributions used in the first stage. As such, stage two involves estimating of 80 GARCH models. For the want of space, the full results of the best distribution are presented². Interestingly, GED retained its status as the most preferred distribution for the GARCH estimation. In other words, irrespective of whether we are considering the pre-test distribution (residuals) or the GARCH-type model, the penchant of GED, over other distributions remains unchanged. As such, our results support the new clarion call in the literature that have argued that the assumption of normal distribution of any series should not be given as true without proper statistical tests (see Aparicio and Estrada, 2001; Harris and Kucukozmen, 2003; Rachev et al. 2008). Table 3 has these results.

The extant literature has shown that return series are sensitive to changes in data frequencies. Hence, the above analyses are replicated for Weekly and monthly frequencies. We observe that there is no change in the position of the preference of GED as the best fit distribution for these additional frequencies.

4. Conclusion

The study provides a new procedure determining the distribution property of stock returns using the G7 stock markets as a case study. It shows that pre-testing the residual of the series for the distribution properties helps to determine the accompanying appropriate distribution for the GARCH error regardless of the choice of GARCH-type model. Results show that GED is the most preferred distribution for all the GARCH-type models considered and are robust to alternative frequencies. Including the suggested pre-test in the list of various preliminary analyses for stock returns will further enrich empirical analyses on the subject and offer a more standard procedure of determining appropriate distribution for GARCH analyses.

² For other distributions, we provide the estimated coefficients for daily and weekly frequencies in the appendix section. The monthly set result can be made available on request.

Table 2: Preliminary Analysis of GARCH Effect

	US		Canada		Germany		France		Japan		UK		Italy	
Type of Test	LL	AIC	LL	AIC	LL	AIC	LL	AIC	LL	AIC	LL	AIC	LL	AIC
Gaussian	13660	-27313	13076	-26145	12460	-24914	12612	-25217	12360	-24714	146311	-28616	10368	-20731
Normal														
Student-T	14287	-28565	13603	-27197	12833	-25658	13046	-26085	12600	-25193	14858	-29708	10785	-21562
Generalised	18509	-37010	17928	-35849	17436	-34863	17591	-35174	17217	-34426	19197	-38385	14489	-28969
Error														
Cauchy	14019	-28032	13025	-26405	12438	-24870	12646	-25286	12101	-24196	14467	-28928	10479	-20951
Laplace	14287	-28549	13550	-27095	12822	-25639	13021	-26036	12552	-25097	14801	-29597	10758	-21510
Logistic	14109	-28211	13490	-26973	12757	-25508	12954	-25903	12751	-25136	14731	-29455	10685	-21364
Hansens	14296	-28581	13612	-27214	12838	-25666	13051	-26092	12603	-25197	14874	-29737	10795	-21579
Skew-t														
Gram-Charlier	11958	-23906	13905	-27799	13625	-27240	10215	-20420	10778	-21547	13794	-27578	11132	-22254
Rayleigh	13975	-27943	13325	-26644	12629	-25251	12966	-25926	12561	-25116	14714	-29422	10680	-21354
Extreme	12268	-24530	118849	-23693	11381	-22737	11357	-22708	11623	-23240	13610	-27194	9458	-18910
Value														
Distribution														

Source: Authors' computation

Table 3: Property Distribution of Bitcoin using various types of GARCH Estimations GED

Country	Stat.	GARCH	GJR	EGARCH	NARCH	NGARCH	AGARCH	APGARCH	NAGARCH	
France	Daily	LL	18749	18800	18794	18756	18560	18819	18742	18596
		AIC	-37483	-	-37572	-37496	-37104	-37621	-37467	-37176
	Weekly	LL	3055	3106	3103	3055	3042	103	3100	3071
		AIC	-6095	-6197	-6190	-6093	-6067	-6189	-6181	-6125
	Monthly	LL	568	583	588	568	568	568	591	586
		AIC	-1122	-1150	-1161	-1120	-1120	-1120	-1163	-1155
Japan	Daily	LL	18511	18578	18565	18509	18355	18592	17174	18390
		AIC	-37007	-37140	-37113	-37002	-36694	-37169	-34330	-36764
	Weekly	LL	3007	3050	3049	3007	2995	3055	3046	3033
		AIC	-5999	-6084	-6083	-5998	-5974	-6094	-6074	-6050
	Monthly	LL	528	542	545	529	523	547	454	541
		AIC	-1043	-1068	-1075	-1041	-1030	-1078	-1072	-1066
UK	Daily	LL	20219	20243	20240	20122	20066	20270	20157	19454
		AIC	-40424	-40471	-40465	-40228	-4011	-40523	-40296	-38893
	Weekly	LL	3193	3194	3211	3193	3180	3195	3187	3162
		AIC	-6371	-6371	-6406	-6371	-6343	-6374	-6356	-6308
	Monthly	LL	548	549	556	548	537	551	549	545
		AIC	-1081	-1081	-1096	-1080	-1058	-1085	-1081	-1074
US	Daily	LL	19916	20057	20037	19917	19709	20080	19990	19766
		AIC	-39818	-40098	-40057	-39817	-39402	-40145	-39962	-39516
	Weekly	LL	3393	3421	3430	3386	3282	3432	3420	3405
		AIC	-6771	-6826	-6844	-6756	-6549	-6847	-6822	-6793
	Monthly	LL	661	675	673	662	658	675	676	669
		AIC	-1308	-1334	-1330	-1307	-1300	-1333	-1333	-1323
Canada	Daily	LL	19288	19353	19365	19295	19035	19362	18919	12628
		AIC	-38563	-38690	-38714	-38574	-38053	-38709	-37820	-25239
	Weekly	LL	3184	3208	3219	3188	3146	3200	3193	3170
		AIC	-6355	-6401	-6422	-6360	-6277	-6384	-6368	-6325
	Monthly	LL	578	585	582	579	577	588	587	582
		AIC	-1142	-1147	-1148	-1142	-1139	-1159	-1156	-1148
Italy	Daily	LL	15502	15535	15517	15511	15316	15560	15442	15359
		AIC	-30990	-31055	-31017	-31006	-30616	-31103	-30866	-30702
	Weekly	LL	2534	2572	2569	2534	2515	2575	2579	2542
		AIC	-5053	-5127	-5122	-5051	-5013	-5133	-5141	-5067
	Monthly	LL	453	464	471	453	450	473	462	469

		AIC	-893	-913	-926	-891	-883	-931	-907	-922
Germany	Daily	LL	18511	18578	18565	18509	18355	18592	17174	18390
		AIC	-37007	-37140	-37113	-37002	-36694	-37169	-34330	-36764
	Weekly	LL	3007	3050	3049	3007	2995	3055	3046	3033
		AIC	-5999	-6084	-6083	-5998	-5974	-6094	-6074	-6050
	Monthly	LL	528	542	545	529	523	547	545	541
		AIC	-1043	-1068	-1075	-1041	-1030	-1078	-1072	-1066

Source: Authors' computation

Note: The structure of the GARCH specifications is (1,1,0). AIC implies Akaike Information Criteria.

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Appendix: Property Distribution of Bitcoin using various types of GARCH Estimations

Appendix1: Gaussian Normal

Country		Stat.	GARCH	GJR	EGARCH	NARCH	NGARCH	AGARCH	APGARCH	NAGARCH
France	Daily	LL	13459	13510	13504	13456	13313	13509	13423	13365
		AIC	-26906	-27005	-26993	-26915	-26611	-27005	-26830	-26715
	Weekly	LL	1997	2020	2018	1998	1986	2019	2018	2002
		AIC	-3982	-4026	-4023	-3983	-3958	-4024	-4019	-3990
Japan	Daily	LL	12817	12842	12842	1289	12771	12852	12822	11446
		AIC	-25622	-25672	-5671	-25624	-25528	-25690	-25629	-22877
	Weekly	LL	2079	2083	2084	2080	2076	2085	2088	2087
		AIC	-4145	-4151	-4153	-4146	-4137	-4156	-4159	-4160
UK	Daily	LL	15102	15121	15116	15103	15000	15128	15058	14994
		AIC	-30192	-30227	-30218	-30191	-29985	-30242	-30099	-29974
	Weekly	LL	2153	2153	2164	2153	2143	2155	2148	2146
		AIC	-4293	-4293	-4313	-4291	-4272	-4295	-4279	-4278
US	Daily	LL	14763	14859	14854	14763	14606	14861	14811	8217
		AIC	-29513	-29704	-29695	-29512	-29198	-29708	-29606	-16421
	Weekly	LL	2322	2342	2348	2321	2277	2348	2342	2331
		AIC	-4632	-4670	-4682	-4627	-4541	-4682	-4668	-4649
Canada	Daily	LL	14059	14089	14085	14060	13898	14090	14004	5900
		AIC	-28106	-28164	-28156	-28105	-27783	-28165	-27992	-11785
	Weekly	LL	2109	2118	2119	2110	2087	2114	2106	2099
		AIC	-4207	-4223	-4224	-4205	-4160	-4215	-4197	-4184
Italy	Daily	LL	11134	11163	11142	11136	-21993	11168	11092	4329
		AIC	-22255	-22311	-22271	-22257	-21993	-22321	-22167	-8644
	Weekly	LL	1643	1656	1649	1643	1617	1656	1661	1639
		AIC	-3274	-3297	-3285	-3272	-3219	-3298	-3305	-3264
Germany	Daily	LL	13256	13308	13299	13256	13136	13308	13255	13157
		AIC	-26500	-26601	-26584	-26498	-26257	-2660	-26494	-26300
	Weekly	LL	1939	1961	1957	1940	1931	1963	1957	1949
		AIC	-3867	-3907	-3901	-3865	-3848	-3911	-3899	-3885

Source: Authors' computation

Note: The structure of the GARCH specifications is (1,1,0). AIC implies Akaike Information Criteria

Appendix 2: Student T

Country		Stat.	GARCH	GJR	EGARCH	NARCH	NGARCH	AGARCH	APGARCH	NAGARCH
France	Daily	LL	13516	13564	13560	13522	13421	13557	13517	12231
		AIC	-27017	-27111	-27104	-27028	-26826	-27097	-27016	-24447
	Weekly	LL	2024	2036	2031	2023	2014	2034	2033	2023
		AIC	-4035	-4055	-4046	-4030	-4012	-4053	-4049	-4031
Japan	Daily	LL	12874	12892	12891	12626	12837	12899	12877	5880
		AIC	-25733	-25767	-25765	-25236	-25658	-25781	-25737	-11745
	Weekly	LL	2107	2108	2103	2107	2104	2108	2101	2108
		AIC	-4199	-4199	-4189	-4198	-4193	-4200	-4185	-4199
UK	Daily	LL	15228	15239	15236	15228	15170	15239	15176	15174
		AIC	-30442	-30463	-30457	-30440	-30323	-30462	-30333	-30322
	Weekly	LL	2196	2197	2201	2196	2190	2198	2165	2192
		AIC	-4378	-4378	-4385	-4376	-4364	-4381	-4313	-4369
US	Daily	LL	14859	14933	14933	14288	14737	14925	14889	14764
		AIC	-29705	-29848	-29850	-28559	-29458	-29833	-29761	-29512
	Weekly	LL	2343	2358	2362	2343	2331	2363	2360	2349
		AIC	-4672	-4700	-4709	-4671	-4645	-4710	-4702	-4683
Canada	Daily	LL	14099	14122	14114	13602	13972	14121	14045	6652
		AIC	-28185	-28228	-28212	-27188	-27928	-28225	-28071	-13289
	Weekly	LL	2127	2131	2129	2127	2115	2129	2123	2121
		AIC	-4239	-4246	-4242	-4237	-4214	-4242	-4227	-4225
Italy	Daily	LL	11191	11221	11201	10811	11107	11218	11090	11124
		AIC	-22368	-22426	-22385	-21605	-22197	-22421	-22161	-22232
	Weekly	LL	1663	1666	1657	1660	1646	1666	1655	1652
		AIC	-3311	-3316	-3298	-3305	-3275	-3316	-3292	-3287
Germany	Daily	LL	13314	13359	13350	13315	13216	13353	13306	13233
		AIC	-26614	-26703	-26633	-26685	-26416	-26690	-26595	-26449
	Weekly	LL	1961	1975	1968	1964	1954	1973	1966	3913
		AIC	-3908	-3934	-3919	-3912	-3893	-3929	-3915	-3874

Source: Authors' computation

Note: The structure of the GARCH specifications is (1,1,0). AIC implies Akaike Information Criteria

Appendix 3: Cauchy

Country		Stat.	GARCH	GJR	EGARCH	NARCH	NGARCH	AGARCH	APGARCH	NAGARCH
France	Daily	LL	12862	12886	12878	12650	12805	12881	12867	12679
		AIC	-25711	-25757	-25742	-25286	-25597	-25748	-25718	-25343
	Weekly	LL	1898	1901	1892	1898	1888	1899	1894	1892
		AIC	-3783	-3788	-3771	-3782	-3763	-3784	-3771	-3770
Japan	Daily	LL	12219	12223	12217	12221	12143	12227	12218	12206
		AIC	-24425	-24432	-24420	-24428	-24272	-24441	-24421	-24398
	Weekly	LL	1980	1980	1974	1980	1977	1980	1979	1978
		AIC	-3948	-3947	-3934	-3947	-3940	-3947	-3942	-3943
UK	Daily	LL	14633	14637	14633	14494	14610	14640	14619	5225
		AIC	-29254	-29260	-29251	-28973	-29207	-29266	-29223	-10435
	Weekly	LL	2083	2083	2084	2083	2078	2084	2067	2075
		AIC	-4154	-4512	-4154	-4152	-4142	-4154	-4118	-4135
US	Daily	LL	14321	14359	14353	14031	14257	14352	14332	14056
		AIC	-28630	-28704	-28691	-28048	-28500	-28690	-28649	-28098
	Weekly	LL	2221	2228	2228	2221	2214	2229	2230	2193
		AIC	-4429	-4442	-4441	-4428	-4414	-4445	-4443	-4372
Canada	Daily	LL	13413	13419	13405	13221	13375	13418	13381	4901
		AIC	-26814	-26824	-26795	-26429	-26737	-26822	-26747	-9788
	Weekly	LL	2005	2008	2002	2004	2001	2005	1998	1984
		AIC	-3998	-4001	-3989	-3995	-3989	-3997	-3979	-3954
Italy	Daily	LL	10655	10670	10655	10655	10625	10669	10647	10635
		AIC	-21298	-21326	-21296	-21297	-21235	-21324	-21278	-21257
	Weekly	LL	1554	1555	1547	1554	1548	1554	1551	1547
		AIC	-3095	-3095	-3080	-3094	-3081	-3095	-3086	-3081
Germany	Daily	LL	12668	12686	12678	12442	12620	12685	12660	12632
		AIC	-25323	-25358	-25341	-24871	-25226	-25356	-25304	-25249
	Weekly	LL	1831	1837	1828	1832	1824	1833	1779	1807
		AIC	-3650	-3660	-3642	-3650	-3633	-3652	-3542	-3600

Source: Authors' computation

Note: The structure of the GARCH specifications is (1,1,0). AIC implies Akaike Information Criteria

Appendix 4: Extreme Value Distribution

Country		Stat.	GARCH	GJR	EGARCH	NARCH	NGARCH	AGARCH	APGARCH	NAGARCH
France	Daily	LL	13011	13038	13039	13045	12971	13018	13004	12474
		AIC	-26010	-26062	-26064	-26077	-25928	-26022	-25992	-24934
	Weekly	LL	2000	2004	2004	2000	1989	2005	1992	1968
		AIC	-3987	-3993	-3994	-3985	-3963	-3995	-3967	-3922
Japan	Daily	LL	12567	12572	12546	12568	12440	12571	123550	12440
		AIC	-25122	-25130	-25077	-25121	-24866	-25128	-24694	-24865
	Weekly	LL	2035	2038	2038	2035	2030	2030	2017	2009
		AIC	-4058	-4061	-4061	-4056	-4045	-4046	-4019	-4003
UK	Daily	LL	14752	14787	14768	14754	14561	14474	14592	14632
		AIC	-29493	-29560	-29523	-29494	-29109	-29534	-29169	-29249
	Weekly	LL	2157	2164	2183	2134	2137	2162	2144	2146
		AIC	-4303	-4314	-4353	-4255	-4259	-4311	-4272	-4279
US	Daily	LL	14653	14653	14645	14655	14443	14653	14476	6164
		AIC	-29293	-29292	-29275	-29296	-28871	-29292	-28933	-12314
	Weekly	LL	2360	2361	2360	2361	2341	2361	2341	2335
		AIC	-4709	-4708	-4706	-4707	-4668	-4709	-4666	-4656
Canada	Daily	LL	13855	13856	13848	13857	13356	13857	13390	12822
		AIC	-27699	-27698	-27682	-27699	-26699	-27701	-26764	-25629
	Weekly	LL	2127	2124	2126	2124	2082	2128	2083	2092
		AIC	-4241	-4233	-4238	-4235	-4149	-4242	-4150	-4169
Italy	Daily	LL	10850	10853	10862	10853	10793	10863	10591	4746
		AIC	-21687	-21693	-21709	-21691	-21571	-21712	-21165	-9478
	Weekly	LL	1667	1667	1658	1667	1643	1664	1640	1638
		AIC	-3321	-3321	-3303	-3320	-3272	-3314	-3264	-32661
Germany	Daily	LL	12879	12880	12888	12881	12603	12895	12796	7093
		AIC	-25745	-25747	-25761	-25748	-25192	-25776	-25576	-14172
	Weekly	LL	1957	1959	1949	1957	1940	1958	1954	1938
		AIC	-3901	-3904	-3884	-3899	-3866	-3902	-3892	-3863

Source: Authors' computation

Note: The structure of the GARCH specifications is (1,1,0). AIC implies Akaike Information Criteria

Appendix 5: Rayleigh

Country		Stat.	GARCH	GJR	EGARCH	NARCH	NGARCH	AGARCH	APGARCH	NAGARCH
France	Daily	LL	12331	13082	9514	12934	12288	13104	13167	5749
		AIC	-24651	-26149	-19014	-25854	-24562	-26193	-26318	-11485
	Weekly	LL	1963	2015	1958	1956	1966	2019	1915	1985
		AIC	-3914	-4016	-3903	-3897	-3918	-4025	-3813	-3955
Japan	Daily	LL	12472	12232	7086	12486	12435	12519	12483	12604
		AIC	-24933	-24450	-14158	-24958	-24856	-25024	-24950	-25194
	Weekly	LL	2088	2101	-1607	2107	2071	1801	2086	2061
		AIC	-4164	-4188	3228	-4199	-4127	-3587	-4157	-4109
UK	Daily	LL	14366	14696	12628	14150	14752	14680	14738	14774
		AIC	-28719	-29379	-25242	28286	-29489	-29346	-29459	-29533
	Weekly	LL	2213	2150	940	2194	2202	2178	2178	2188
		AIC	-4413	-4286	-1865	-4375	-4389	-4343	-4340	-4361
US	Daily	LL	14134	14161	12358	13778	13948	14257	14186	14100
		AIC	-28255	-28308	-24701	-27541	-27881	-28501	-28356	-28185
	Weekly	LL	2269	2245	2248	2267	2042	2285	2205	2225
		AIC	-4526	-4477	-4483	-4520	-4070	-4557	-4394	-4436
Canada	Daily	LL	13591	13459	8261	13254	13524	13755	13281	8983
		AIC	-27169	-26905	-16509	-26494	-27034	-27496	-26547	-17952
	Weekly	LL	2073	2133	2078	2012	2067	2082	2053	2105
		AIC	-4134	-4252	-4141	-4011	-4120	-4150	-4090	-4196
Italy	Daily	LL	10728	9917	2450	10708	10624	10698	10847	10417
		AIC	-21443	-19820	-4886	-21402	-21235	-21382	-21679	-20775
	Weekly	LL	1643	1674	1194	1589	1598	1651	1601	438
		AIC	-3275	-3334	-2374	-3165	-3182	-3288	-3186	-862
Germany	Daily	LL	12637	12950	12772	12400	11291	12775	12968	12396
		AIC	-25262	-25887	-25529	-24785	-22568	-25537	-25921	-24779
	Weekly	LL	1958	1985	1741	1922	1868	1958	1882	1934
		AIC	-3904	-3955	-3468	-3830	-3722	-3903	-3749	-3854

Source: Authors' computation

Note: The structure of the GARCH specifications is (1,1,0). AIC implies Akaike Information Criteria

Appendix 6: Logistic

Country		Stat.	GARCH	GJR	EGARCH	NARCH	NGARCH	AGARCH	APGARCH	NAGARCH
France	Daily	LL	13518	13561	13547	13520	13424	13557	13514	13030
		AIC	-27024	-27108	-17080	-27025	-26834	-27100	-27012	-26046
	Weekly	LL	2021	2033	2029	2023	2012	2032	2030	1982
		AIC	-4030	-4053	-4044	-4031	-4009	-4050	-4045	-3951
Japan	Daily	LL	12868	12885	12878	12574	12826	12892	12873	11983
		AIC	-25725	-25756	-25742	-25135	-25638	-25770	-25729	-23951
	Weekly	LL	2103	2105	2099	2104	2098	2106	2104	2105
		AIC	-4195	-4196	-4184	-4195	-4181	-4197	-4193	-4196
UK	Daily	LL	15224	15236	15227	14716	15158	15237	15192	4841
		AIC	-30435	-30458	-30439	-29417	-30301	-30460	-30368	-9668
	Weekly	LL	2195	2196	2196	2195	2149	2197	2192	2192
		AIC	-4378	-4379	-4377	-4376	-4284	-4381	-4369	-4369
US	Daily	LL	14845	14934	14925	14136	14734	14928	14892	14405
		AIC	-29678	-29854	-29836	-28259	-29455	-29842	-29768	-28796
	Weekly	LL	2343	2359	2360	2344	2332	23663	2360	2283
		AIC	-4675	-4703	-4706	-4673	-4560	-4712	-4706	-4551
Canada	Daily	LL	14093	14112	14096	13499	13960	14112	14042	4596
		AIC	-28174	-28212	-28178	-26985	-27906	-28210	-28069	-9177
	Weekly	LL	2128	2133	2127	2128	2113	2130	2123	2063
		AIC	-4243	-4252	-4239	-4241	-4213	-4246	-4231	-4112
Italy	Daily	LL	11187	11213	11190	10690	11082	11211	11169	11119
		AIC	-22361	-22411	-22366	-21366	-22149	-22408	-22322	-22225
	Weekly	LL	1656	1662	1653	1656	1637	1662	1654	1648
		AIC	-3301	-3310	-3291	-3299	-3261	-3311	-3293	-3281
Germany	Daily	LL	13311	13354	13339	13313	13218	13349	13308	12855
		AIC	-26609	-26695	-26664	-26612	-26423	-26684	-26548	-25695
	Weekly	LL	1960	1970	1964	1959	1931	1971	1967	1961
		AIC	-3909	-3926	-3914	-3904	-3849	-3927	-3918	-3907

Source: Authors' computation

Note: The structure of the GARCH specifications is (1,1,0). AIC implies Akaike Information Criteria

Appendix 7: Laplace

Country		Stat.	GARCH	GJR	EGARCH	NARCH	NGARCH	AGARCH	APGARCH	NAGARCH
France	Daily	LL	13422	13452	13422	13423	13357	13449	13418	5859
		AIC	-26831	-26890	-26871	-26832	-26699	-268884	-26820	-11704
	Weekly	LL	2005	2012	20006	2005	1948	2011	2008	2002
		AIC	-3998	-4010	-3998	-3997	-3882	-4007	-4000	-3990
Japan	Daily	LL	12762	12773	12769	12554	12737	12778	12764	12748
		AIC	-25511	-25531	-25525	-25093	-24560	-25541	-25512	-25481
	Weekly	LL	2088	2089	2084	2089	2084	2090	2090	2089
		AIC	-4165	-4165	-4155	-4164	-4154	-4166	-4164	-4163
UK	Daily	LL	15148	15160	15151	15148	15103	15161	15124	6388
		AIC	-30284	-30305	-30288	-30282	-30191	-30308	-30231	-12762
	Weekly	LL	2182	2183	2185	2182	2169	2184	2177	2179
		AIC	-4353	-43552	-4356	-4351	-4325	-4353	-4338	-4344
US	Daily	LL	14830	14883	14879	14354	14739	14877	14846	14346
		AIC	-29648	-29751	-29743	-28694	-29463	-29740	-29676	-28678
	Weekly	LL	2326	2338	2339	2326	2096	2340	2339	2331
		AIC	-4641	-4661	-4663	-4639	-4177	-4667	-4661	-4647
Canada	Daily	LL	13969	13972	13970	13563	13887	13980	13916	8283
		AIC	-27927	-27931	-27926	-27113	-27761	-27947	-27816	-16552
	Weekly	LL	2112	2116	2111	2113	2103	2114	2109	2108
		AIC	-4213	-4217	-4209	-4212	-4192	-4214	-4203	-4168
Italy	Daily	LL	11109	11127	11110	10758	11048	11126	11031	4732
		AIC	-22206	-22240	-22205	-21501	-22081	-22238	-22046	-9450
	Weekly	LL	1647	1650	1642	1647	1636	1651	1631	1638
		AIC	-3281	-3286	-3270	-3280	-3258	-3288	-3245	-3263
Germany	Daily	LL	13218	13429	13237	13206	13149	13244	13209	7355
		AIC	-26424	-26483	-26459	-26397	-26285	-26475	-26402	-14697
	Weekly	LL	1940	1947	1942	1940	1933	1946	1917	1939
		AIC	-3868	-3879	-3870	-3867	-3852	-3879	-3818	-3863

Source: Authors' computation

Note: The structure of the GARCH specifications is (1,1,0). AIC implies Akaike Information Criteria

Appendix 8: Gram-Charlie

Country		Stat.	GARCH	GJR	EGARCH	NARCH	NGARCH	AGARCH	APGARCH	NAGARCH
France	Daily	LL	15266	15396	15373	13895	11822	12233	14631	15008
		AIC	-30516	-30774	-30728	-27772	-23625	-24449	-29042	-29999
	Weekly	LL	2361	1475	2457	1527	2341	1797	2465	1188
		AIC	-4706	-2931	-4896	-3036	-4664	-3577	-4911	-2359
Japan	Daily	LL	11665	13270	14664	10574	12976	14026	14561	10370
		AIC	-23313	-26523	-29310	-21131	-25934	-28034	-29103	-20721
	Weekly	LL	1779	2431	1295	1848	2400	2335	2452	1398
		AIC	-3542	-4845	-2573	-3679	-4782	-4652	-4884	-2778
UK	Daily	LL	16485	15686	16705	10523	16414	16773	16153	15307
		AIC	-32953	-31354	-33392	-21028	-32809	-33527	-32286	-30596
	Weekly	LL	433	294	401	392	392	377	410	360
		AIC	-850	-571	-785	-766	-767	-736	-801	-701
US	Daily	LL	16418	16067	16619	12356	15682	16679	16540	12880
		AIC	-32820	-32116	-33219	-24693	-31346	-33340	-33060	-25741
	Weekly	LL	2707	2417	1897	2693	2660	2784	2507	2507
		AIC	-5398	-4817	-3776	-5368	-5301	-5550	-4994	-5483
Canada	Daily	LL	10267	16066	16100	13066	15570	16135	16031	15767
		AIC	-20518	-32115	-32181	-26114	-31122	-32253	-32043	-31517
	Weekly	LL	1859	1592	1759	1829	2430	2532	2531	1276
		AIC	-3702	-3166	-3500	-3640	-4842	-5046	-5042	-2534
Italy	Daily	LL	11113	12701	8997	9551	11170	10722	12608	12461
		AIC	-22209	-25384	-17977	-19083	-22322	-21425	-25196	-24904
	Weekly	LL	1266	2039	2033	1463	1937	2051	2029	1354
		AIC	-2515	-4061	-4048	-2908	-3855	-4083	-4039	-2690
Germany	Daily	LL	11330	15124	9438	13816	13276	15188	11738	14181
		AIC	-22643	-30229	-18858	-27615	-26534	-30358	-23455	-28344
	Weekly	LL	1672	2402	2099	1632	2298	2196	2399	2377
		AIC	-3327	-4785	-4181	-3252	-4579	-4375	-4778	-4736

Source: Authors' computation

Note: The structure of the GARCH specifications is (1,1,0). AIC implies Akaike Information Criteria

Appendix 9: Hansen

Country		Stat.	GARCH	GJR	EGARCH	NARCH	NGARCH	AGARCH	APGARCH	NAGARCH
France	Daily	LL	13532	13578	13571	13061	13422	13574	135330	13465
		AIC	27048	-27137	-27124	-26105	-26826	-27129	-27040	-26913
	Weekly	LL	2037	2048	2044	2037	2020	2047	2044	2033
		AIC	-4057	-4079	-4070	-4055	-4022	-4077	-4068	-4047
Japan	Daily	LL	12881	12900	12899	12879	12844	12908	12874	7186
		AIC	-25747	-25782	-25781	-25741	-25669	-25797	-25729	-14354
	Weekly	LL	2110	2111	2106	2110	2102	2112	2113	1701
		AIC	-4204	-4204	-4195	-4203	-4185	-4206	-4205	-3384
UK	Daily	LL	15245	15259	15249	14875	15187	15259	15216	15204
		AIC	-30474	-30500	-30480	-29732	-30356	-30499	-30411	-30390
	Weekly	LL	2207	2207	2212	2207	2187	2209	2202	2202
		AIC	-4397	-4396	-4405	-4395	-4356	-4400	-4384	-4387
US	Daily	LL	14882	14958	14957	14300	14753	14954	14914	11860
		AIC	-29747	-29899	-29896	-28582	-29488	-29890	-29807	-23701
	Weekly	LL	2360	2373	2379	2360	2346	2381	2347	1958
		AIC	-4703	-4728	-4739	-4702	-4675	-4743	-4674	-3898
Canada	Daily	LL	14125	14149	14143	13613	13961	14148	14036	14018
		AIC	-28233	-28280	-28269	-27207	-27904	-28277	-28053	-14018
	Weekly	LL	2146	2149	2149	2146	2132	2146	2140	2137
		AIC	-4276	-4280	-4280	-4274	-4245	-4275	-4260	-4255
Italy	Daily	LL	11208	11236	11214	10819	11097	11234	11124	10802
		AIC	-22400	-22454	-22409	-21620	-22175	-22450	-22228	-21587
	Weekly	LL	1677	1684	1674	1677	1664	1684	1683	1672
		AIC	-3339	-3350	-3330	-3337	-3311	-3350	-3346	-3327
Germany	Daily	LL	13318	13367	13358	13320	133208	13361	13319	5812
		AIC	-26620	-26716	-26697	-26622	-26397	-26703	-26618	-11606
	Weekly	LL	1970	1983	1979	1971	1960	1984	1977	1972
		AIC	-3924	-3948	-3939	-3924	-3903	-3951	-3934	-3926

Source: Authors' computation

Note: The structure of the GARCH specifications is (1,1,0). AIC implies Akaike Information Criteria