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Testing for time-varying stochastic volatility in Bitcoin returns

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Abstract

The study will be the first to offer empirical justification for time-varying stochastic volatility in Bitcoin returns. Specifically, it tests for time variation in both the trend and transitory components of the stochastic volatility using the unobserved components model that accounts for same. Thereafter, it calculates the Bayes factor using the approach of Chan (2018) which involves the Savage-Dickey density ratio in order to avoid the computation of the marginal likelihood. The results overwhelmingly support at least one time-varying stochastic volatility component in Bitcoin returns and the transitory component is favoured in this regard. These results are robust to different data frequencies.

JEL Classification: C11; C53; G17

Key words: Bitcoin returns, Time-varying stochastic volatility, Bayes factor

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1.0 Motivation

No doubt, the behavior of Bitcoin returns is fraught with high risks and uncertainties and by implication high volatilities owing to the sensitivity of the digital currency market to news effect (see Cheah and Fry, 2015; Baur et al., 2018; Corbet et al., 2018; Cheah et al., 2018; Lahmiri and Bekiros, 2018; Lahmiri et al., 2018). This majorly explains the choice of volatility models for analyzing Bitcoin returns (see Katsiampa, 2017 for a review). However, all the analyses involving the return volatility of Bitcoin prices are restricted to the GARCH-type (deterministic) volatility models (see Glaser et al., 2014; Gronwald, 2014; Bouoiyour and Selmi, 2016; Dyhrberg, 2016a,b; Bouri et al., 2017; Katsiampa, 2017). Meanwhile, allowing for stochastic behaviour of return volatility particularly with time-varying parameters if found to exist may produce better forecast results than their GARCH counterparts (see Chan and Grant, 2016 for a review of the literature).

Thus, this study offers the following distinctive contributions as regards Bitcoin market analyses. First, it tests whether time-varying stochastic volatility is necessary whether in the trend or transitory component stochastic volatility of Bitcoin returns. This assumption of time-variation in the coefficients and volatilities is usually taken for granted when modeling and forecasting macroeconomic and financial series. The computational complexities involved in pre-testing for time-variation in stochastic volatility (SV) with multiple state-space models constitute a major disincentive for conducting such test. However, the recent approach proposed by Chan (2018) offers an easy technique to test for time-variation in coefficients and volatilities that circumvents the computation of the marginal likelihood which is non-trivial for nonlinear state space models with multiple states. This approach is particularly useful when computing the Bayes factor for nested models which is the case in this study where one model is a smaller (restricted) version of the other (unrestricted)

model. The former involves the SV model with constant coefficients while the latter allows for time-variation. The Bayes factor is commonly used to compare models and its calculation usually requires the computation of marginal likelihoods which can be very demanding for nonlinear state space models as considered in this study. The approach of Chan (2018) computes the Bayes factor using the Savage-Dickey density ratio which requires only the estimation of the unrestricted models and no explicit computation of the marginal likelihood is needed.

In terms of practical relevance of the study outcomes, the better the refinements introduced into the predictive model of Bitcoin returns, the higher the accuracy of forecast results and by extension, investors and policy makers are better positioned to deal with the associated risks and uncertainties in the Bitcoin market. More so, the contribution in the refinement of modeling framework for Bitcoin returns is particularly important for future policy formulation by relevant authority since the market is not (yet) regulated and not closely supervised or overseen by any public authority in spite of the fact it falls within central banks' responsibility due to shared characteristics with payment systems (European Central Bank, 2015).

The remainder of this study is organized as follows: Section 2 describes the model set-up for the empirical analyses; Section 3 discusses the results while Section 4 concludes the paper.

2.0 Model set-up

As previously mentioned, the time-variation in stochastic volatility is often assumed when modelling and forecast financial and economic series without any formal test to confirm the validity of making such assumption. We begin the analysis by specifying an unobserved components model where the series is decomposed into

transitory and trend components and each component follows an independent stochastic volatility process (see Stock and Watson, 2007; Chan, 2018)¹:

$$\begin{aligned}
 r_t &= \tau_0 + \omega_\tau \tilde{\tau}_t + e^{\frac{1}{2}(h_0 + \omega_h \tilde{h}_t)} \varepsilon_t^r; \\
 \tilde{\tau}_t &= \tilde{\tau}_{t-1} + e^{\frac{1}{2}(g_0 + \omega_g \tilde{g}_t)} \varepsilon_t^\tau; \\
 \tilde{h}_t &= \tilde{h}_{t-1} + \varepsilon_t^h; \\
 \tilde{g}_t &= \tilde{g}_{t-1} + \varepsilon_t^g;
 \end{aligned} \tag{1}$$

where r_t is the log return of bitcoin price computed as $\log(p_t/p_{t-1})$; p_t is the bitcoin price; $\tilde{\tau}_t$ denote the time-varying intercept while \tilde{g}_t and \tilde{h}_t are the time-varying stochastic volatilities in the trend and transitory components respectively. The $\tilde{\tau}_t$, \tilde{g}_t and \tilde{h}_t are further defined as $\tilde{\tau}_t = (\tau_t - \tau_0)/\omega_\tau$, $\tilde{g}_t = (g_t - g_0)/\omega_g$ and $\tilde{h}_t = (h_t - h_0)/\omega_h$ following Fruhwirth-Schnatter and Wagner (2010) and therefore the corresponding state equations are initialized with $\tilde{\tau}_1 \sim N(0, V_\tau \exp(g_0 + \omega_g \tilde{g}_1))$, $\tilde{g}_1 \sim N(0, V_g)$ and $\tilde{h}_1 \sim N(0, V_h)$ respectively. Also, the error terms ε_t^r , ε_t^τ , ε_t^h and ε_t^g are independent and follow Gaussian distribution; while the parameters to be estimated are τ_0 , h_0 , g_0 , ω_τ , ω_g and ω_h ; the variances V_τ , V_h and V_g are known constants and for the purpose of empirical application, they are fixed at 10 which is the same value in Stock and Watson (2007) and Chan (2018). Equally, we assume normal priors for ω_τ , ω_g and ω_h where $\omega_\tau \sim N(0, V_{\omega_\tau})$, $\omega_g \sim N(0, V_{\omega_g})$ and $\omega_h \sim N(0, V_{\omega_h})$ and set V_{ω_τ} , V_{ω_g} and V_{ω_h} at 0.2 as in the latter studies.

Since we wish to test for time-variation in the stochastic volatilities, then, we need to consider two nested models: the unrestricted model (where the variances are time-varying as indicated in equation (1); i.e., $\omega_h^2 \neq 0$ and $\omega_g^2 \neq 0$) and the restricted model (where the variances are constant; $\omega_h^2 = 0$ and $\omega_g^2 = 0$). Let us define the unrestricted

¹ The unobserved components model specified here is the non-centred parameterization variant proposed by Chan (2018) which overcomes the difficulties associated with the indirect calculation of the Bayes factor on the basis of the marginal likelihood.

model as in equation (1) as M_1 and the restricted version as M_2 . Typically, the Bayes factor is prominently used to compare nested models involving time-varying parameters and is therefore considered in this study. The Bayes factor can be specified as: $BF_{12} = p(r|M_1)/p(r|M_2)$ where $p(r|M_i)$ is the marginal data density under model M_i evaluated at the observed data r . As noted earlier, the calculation of the Bayes factor follows the Chan (2018) which involves the Savage-Dickey density ratio as specified below²:

$$BF_{uh} = p(\omega_h = 0) / p(\omega_h = 0 | r) \quad (2)$$

$$BF_{ug} = p(\omega_g = 0) / p(\omega_g = 0 | r) \quad (3)$$

$$BF_{u,hg} = p(\omega_h = \omega_g = 0) / p(\omega_h = \omega_g = 0 | r) \quad (4)$$

where the subscript u is for the unrestricted model and note that $\omega_h \neq 0$ in this case; h is for the restricted model where the transitory component is fixed, that is, $\omega_h = 0$; g is another variant of the restricted model the trend component is also fixed, that is, $\omega_g = 0$;³ and the combined subscripts hg imply the simultaneous restriction of both the transitory and trend components to zero, that is, $\omega_h = \omega_g = 0$. The corresponding Bayes factors are respectively denoted as BF_{uh} ; BF_{ug} and $BF_{u,hg}$.⁴ If the $BF > 1$, regardless of the case being considered, it then follows that the unrestricted model is more representative of the observed data in question; otherwise, the restricted model will produce a better fit. A statistically significant estimate of the Bayes factor implies the preference of the unrestricted model over the restricted version; while the reverse is the case if it is not significant.

3.0 Results and Discussion

² An alternative approach is to compute the marginal likelihood for the competing models however this approach is computationally demanding as it requires evaluating the integrated likelihood which implies that integrating out all the states (Chan, 2018).

³ The unrestricted models being referred to in (2) and (3) are special cases of equation (1) where the time-varying volatility is singly captured for transitory and trend components and not both in the same volatility framework.

⁴ The computational details of the Bayes factor are provided in Chan (2018).

The study utilizes daily, weekly and monthly data frequencies of Bitcoin returns for the period July 19, 2010 – May 03, 2018 and all the data are sourced from the Bloomberg terminal. The estimates of the log Bayes factors for the different data frequencies are presented in Table 1. Recall that if $BF > 1$, then the unrestricted model of time-varying volatility is necessary when modelling and forecasting Bitcoin returns. However, since we are using the log Bayes factor, then, the unrestricted model is only preferred if $BF > 0$; otherwise, the time-varying parameter assumption may not be appropriate. On the basis of these decision criteria, we find evidence in support of time varying parameter volatility in the transitory component since the $\log BF_{uh}$ values are positive while the fixed coefficient assumption is more suitable for the trend component as the $\log BF_{ug}$ is negative. As a confirmatory test, the results of the $\log BF_{u,hg}$ which jointly tests for the two time-varying stochastic volatility components, equally suggest the need to account for at least one of the two components. This evidence is consistent across the three data frequencies, thus, modelling Bitcoin returns with time-varying stochastic volatility may produce better forecast performance than models that ignore same. Another striking observation from the results is that statistically significant log of Bayes factors tend to decline with lower frequency; in other words, the higher the frequency, the more likely is the unrestricted model than the restricted model over the given data.

The figures showing the behaviour of the two components depict more time-variation in the standard deviations of the transitory components while the trend component seems to be fairly stable over time (see Figures 1 – 3). This further reinforces the results reported in Table 1 and also justifies the consideration of the SV-type models for modelling the behaviour of Bitcoin returns.

Table 1: The results of the log Bayes factors the three data frequencies

	Daily	Weekly	Monthly
$\log BF_{uh}$	2430.5 (22.11)	415.6 (5.89)	40.9 (2.27)
$\log BF_{ug}$	-1.3 (0.89)	-2.0 (0.15)	-0.3 (0.47)
$\log BF_{u,hg}$	2481.7 (16.22)	444.5 (6.75)	51.3 (2.80)

Note: The numerical standard errors are reported in parentheses.

Figure 1: Time-varying standard deviation of transitory $\exp(h_t/2)$ and trend $\exp(g_t/2)$ components for Bitcoin returns

Figure 1A: For daily frequency

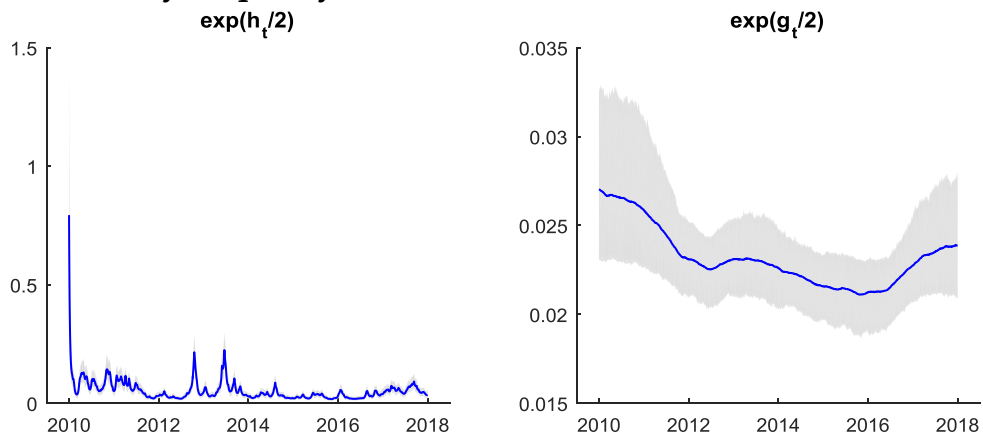


Figure 1B: For weekly frequency

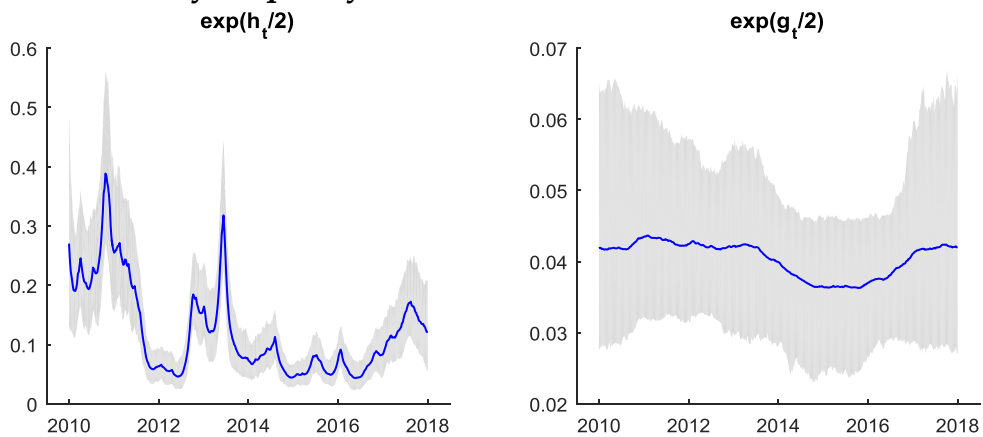
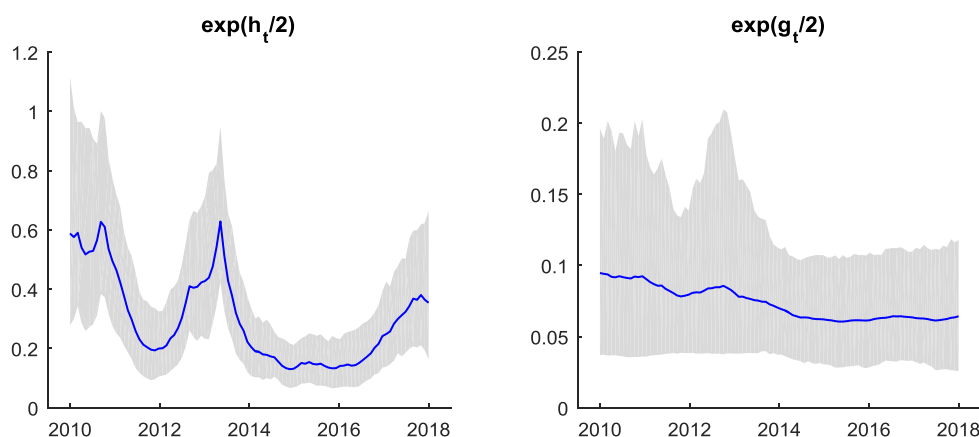


Figure 1C: For monthly frequency



4.0 Conclusion

In this study, we provide empirical justification for time-varying stochastic volatility in Bitcoin returns. This is achieved by formally testing for time-variation in both the trend and transitory components in a typical unobserved component model with stochastic volatility. The results obtained resoundingly support at least one stochastic volatility component when modelling Bitcoin returns and the transitory component seems to be favoured in this regard. The results are insensitive to data frequency and therefore modelling Bitcoin returns with time-varying stochastic volatility may produce better forecast results than those that tend to ignore same.

References

- Baur, D.G., Hong, K. and Adrian, D.L. (2018). Bitcoin: Medium of Exchange or speculative assets? *Journal of International Financial Markets, Institutions and Money*, 54: 177-189.
- Bouoiyour, J. and Selmi, R. (2016). Bitcoin: a beginning of a new phase? *Economics Bulletin*, 36(3), 1430-1440.
- Bouri, E., Azzi, G. and Dyrberg, A. H. (2017). On the return-volatility relationship in the Bitcoin market around the price crash of 2013. *Economics*, 11 (2), 1-16.
- Chan, J.C.C. (2018). Specification Tests for Time-Varying Parameter Models with Stochastic Volatility. *Econometric Reviews*, 37(8), 807-823.
- Chan, J.C.C. and Grant, A.L. (2016). Modeling Energy Price Dynamics: GARCH versus Stochastic Volatility. *Energy Economics*, 54, 182-189.
- Cheah, E., Mishra, T., Parhi, M. and Zhang, Z. (2018). Long memory interdependency and inefficiency in Bitcoin markets. *Economics Letters*, 167: 18-25.

- Cheah, E.T. and Fry, J.M. (2015). Speculative bubbles in Bitcoin markets? An empirical investigation into the fundamental value of Bitcoin. *Economics Letters*, 30: 32-36.
- Corbet, S., Lucey, B. and Yarovaya, L. (2018). Datestamping the Bitcoin and Ethereum bubbles. *Finance Research Letters*, In Press, DOI: 10.1016/j.frl.2017.12.006.
- Dyhrberg, A. H. (2016a). Bitcoin, gold and the dollar - A GARCH volatility analysis. *Finance Research Letters*, 16, 85-92.
- Dyhrberg, A. H. (2016b). Hedging capabilities of Bitcoin. Is it the virtual gold? *Finance Research Letters*, 16, 139-144.
- Katsiampa, P. (2017). Volatility estimation for Bitcoin: A comparison of GARCH models. *Economics Letters*, 158, 3-6.
- Lahmiri, S. and Bekiros, S. (2018). Chaos, randomness and multi-fractality in Bitcoin market. *Chaos, Solitons and Fractals*, 106: 28-34.
- Lahmiri, S., Bekiros, S. and Salvi, A. (2018). Long-range memory, distributional variation and randomness of bitcoin volatility. *Chaos, Solitons and Fractals*, 107: 43-48.
- Stock, J. H. and Watson, M. W. (2007). Why has U.S. inflation become harder to forecast? *Journal of Money Credit and Banking*, 39(s1), 3-33.