Working Paper Series: 17 Jan/2015

Modelling Oil price-US stocks nexus: A VARMA- BEKK-AGARCH Approach

Afees A. Salisu and Tirimisiyu F. Oloko

Centre for Econometric and Allied Research (CEAR), University of Ibadan, Nigeria.
www.cear.org.ng

Modelling Oil price-US stocks nexus: A VARMA- BEKK- AGARCH Approach

Afees A. Salisu[a,b,c] and Tirimisiyu F. Oloko[b]

[a] Department of Economics, University of Ibadan, Nigeria.
[b] Centre for Econometric and Allied Research (CEAR), University of Ibadan, Nigeria.
Email: olokotrimicy@yahoo.com; tf.oloko@cear.org.ng
[c] Corresponding author: Department of Economics and Centre for Econometric and Allied Research, University of Ibadan, Oyo State, Nigeria.
Email: aa.salisu@mail.ui.edu.ng; aa.salisu@cear.org.ng
Mobile: (+234) 8034711769

Abstract

This study adds to the existing literature on oil price-US stocks nexus in three ways. First, it employs the VARMA-AGARCH model developed by McAleer et al. (2009) within the context of BEKK framework using West Texas Intermediate (WTI) and Brent as proxies for oil market and S&P stocks as a proxy for US stock market. Secondly, it modifies the model to include endogenously determined structural break using the general structure for analyzing breaks with unit roots in Perron (2006). Third, it uses the adopted model to compute optimal portfolio weight and hedge ratios between oil price and US stocks using different sample data based on the break date. On average, our empirical evidence suggests a significant positive return spillover from US stock market to oil market and bi-directional shock spillovers between the two markets. In addition, there is significant own asymmetric shock effect in both markets while volatility spillover from oil market to stock market became pronounced after the break which coincides with the period of global economic slowdown. Similarly, the results of portfolio management differ across the sample data. More importantly, we find that ignoring structural break when it exists may exaggerate hedging effectiveness.

Keywords: Oil price, S&P stocks, VARMA-BEKK-AGARCH, Spillover effect, Asymmetric effect, Portfolio management

JEL Classification: C3; G12, Q43

Modelling Oil price-US stocks nexus: A BEKK-VARMA-AGARCH Approach

1.0 Introduction

The increasing interest in oil market research is not unexpected. First, oil has remained a major source of world energy accounting for about 34 percent of the world's energy needs (IEA, 2008). The International Energy Agency (IEA) projects that oil will provide 30 percent of the world's energy mix in 2030.[1] Second, oil price is susceptible to high volatility due to supply shocks and therefore, the risk and uncertainties occasioned by oil price volatility usually affect investors' portfolios particularly, portfolio managers seeking to make optimal portfolio allocations (see Arouri, et al., 2011a,b for a survey of the literature). Third, oil can affect the macro-economy fundamentals which have to be addressed by the policy makers (see Sadorsky, 1999; Arouri and Nguyen, 2010). For example, rising oil prices are often considered to be inflationary by policy makers and central banks respond to inflationary pressures by raising interest rates which affect the discount rate used in the stock pricing formula (see Basher, et al., 2012). Also, rising oil prices affect both the consumers and the producers. The consumers are affected in form of higher prices on final goods and services (see Faff and Brailsford, 1999) while the producers are confronted with lower demand for final goods and services which consequently reduces profits and lowers the scale of operations of the affected firms. More importantly, as the demand for oil continues to increase and its utilization becomes more inevitable both at commercial and household levels, understanding the complex relationships between oil and the rest of the economy will become even more important. At best, uncertainties including unanticipated shocks may become more prevalent, and understanding their possible implications will be of interest to both investors and policy makers.

[1] International Energy Agency (IEAi, World Energy Outlook 2008.

Therefore, all the stakeholders in the energy market, oil market in particular, are usually concerned with how oil price returns and volatility are transmitted across the key sectors of the economy namely financial and real sectors. By implication, analysis of the spillover effects of oil price provides useful insights into how the fluctuations of oil prices will affect economic activities including portfolios of profit maximizing investors. In fact, the recent uncertainty that has characterized oil prices has continued to generate a lot of interests in the relationship among oil prices, financial markets and the economy. Thus, this paper focuses on the spillover effects between crude oil price and stock market.

The theoretical underpinning for the relationship between oil price and stock returns is usually premised on the fact that oil prices can affect stock prices directly by impacting on future cash flows or indirectly through an impact on the interest rate used to discount the future cash flows (see Basher et al., 2012). This area was empirically pioneered by Huang et al. (1996) and Jones and Kaul (1996) with further contributions by Faff and Brailsford (1999) and Sadorsky (1999). Recent papers excluding those on the US (in the last ten years or so) include, but not limited to, Hammoudeh and Aleisa (2004), Hammoudeh and Li (2005), El-Sharif et al. (2005), Jimenez-Rodriguez and Sanchez (2005), Agren (2006), Basher and Sadorsky (2006), Boyer and Filion (2007), Malik and Hammoudeh (2007), Cong et al. (2008), Henriques and Sadorsky (2008), Apergis and Miller (2009), Bjornland (2009), Narayan and Narayan (2010), Fayyad and Daly (2011), Filis et al. (2011), Arouri et al. (2011a,b and 2012), Fowowe (2013), Wang et al. (2013) and Jouini and Harrathi (2014). The findings however have been mixed and inconclusive.

In the present paper, our focus is on oil price and US stock returns. Interestingly, the US seems to have received a great deal of attention among researchers in relation to the subject than any other country or region in the world to the extent that one may assume that the issue has been exhaustively debated in the literature. One of the attractions to US is the fact that financial experts/analysts/commentators have continued to highlight the interdependence between the US stock market and oil price shocks which necessitate further empirical explanations. For instance, the New York Times (in June 11, 2008) headlines that US stocks plunge after oil climbs by $6" and in a similar fashion, the Wall Street Journal (in August 8, 2008) captions that US stocks rally after crude drops to 3-month low (Alsalman and Herrera, 2013). Also, the *Financial Times* on August 21, 2006, attributes the decline of the US stock market to an increase in crude oil prices caused by concerns about the political stability in the Middle East (including the Iranian nuclear program, the fragility of the ceasefire in Lebanon, and terrorist attacks by Islamic militants) (Kilian and Park, 2009). In addition, unlike most economies, there has been increasing evidence suggesting that oil price changes should not be considered exogenous with respect to US and global macroeconomic conditions (see Barsky and Kilian, 2002, 2004; Kilian, 2008, 2009; and Conrad et al., 2014) and therefore, joint dynamics between oil and macroeconomic variables including stock returns can be explored within the US context. Among the recent papers focusing on US are Hammoudeh et al. (2004), Ghouri (2006), Park and Ratti (2008), Kilian and Park (2009), Elyasiani et al. (2012), Fan and Jahan-Parvar (2012), Alsalman and Herrera (2013), Mollick and Assefa (2013), Conrad et al. (2014), and Kang et al. (2014). For example, Ghouri (2006) finds an inverse relationship between oil price (using West Texas Intermediate (WTI)) and US oil stocks while Kilian and Park (2009) demonstrate that the reaction of US real stock returns to an oil price shock differs greatly depending on whether the change in the price of oil is driven by demand or supply shocks in the oil market. They find that positive oil-specific demand shocks lead to higher stock prices. The Kilian and Park (2009) argument was revisited by Kang et al. (2014) when they examine the effect of the demand and supply shocks driving the global crude oil market on aggregate US bond index real returns. They however find that a positive innovation in aggregate demand has a negative effect on US real bond return.[2]

Notwithstanding the depth of research on oil price and US stocks, this study adds to the literature in the following ways. First, it employs a vector autoregressive moving average – asymmetric generalized conditional heteroscedasticity (VARMA-AGARCH) model developed by McAleer et al. (2009) implemented within the context of BEKK framework. This model allows for the estimation of returns, volatility and shock spillovers as well as asymmetric effects. Unlike the CCC and DCC frameworks that only incorporate own-market asymmetric effects, the BEKK version captures both own-market and cross-market asymmetric effects. Thus, in addition to the fact that the model deals with effects of news (good or bad news) in each market on own volatility, we are also able to evaluate how news in one market (the crude oil market, for example) may fuel higher (lower) volatility in the other market (say stock market). Second, the model is modified to include structural breaks determined endogenously using Perron (2006) unit root test with structural breaks. This modification is necessary as most financial series are susceptible to random walk and structural breaks and therefore, ignoring them when they are present may yield invalid inferences (see Andreou and Ghysels, 2009). In addition, we estimate the CCC and DCC variants and thereafter, we compare their results with the BEKK's version in order to ascertain the robustness of the latter.

The third contribution relates to the use of the adopted model to compute optimal portfolio weight and hedging ratios between oil price and US stocks. This is particularly useful to investors/portfolio managers who may be interested in maximizing their returns or hedging their risks attributable to an asset without reducing the returns of other assets. Finally, this study also

[2] A more comprehensive literature review is provided in section 2.

provides a review that reveals notable differences among the existing studies on US. These differences stem from the methodological approaches, sample data and proxies for oil price and US stocks adopted by the existing studies and all these put together justify the reported mixed findings in the literature.

The remainder of the paper is organized as follows. Section 2 provides literature review covering majorly studies on US in order to offer insights into the different dimensions that have evolved in the analysis of oil price-stock market nexus and also tease out the need for further research. Section 3 provides preliminary analyses while the estimable model is described in section 4 and section 5 discusses the estimation results. In section 6, the optimal portfolio management between oil price and US stock returns is analysed while section 7 concludes the paper.

2.0 Literature Review

The analyses of the relationship between oil price and US stock returns have been well documented in the literature. The empirical explanation for this nexus was pioneered by Huang et al. (1996) and Jones and Kaul (1996). Huang et al. (1996) investigate the dynamic interactions between oil futures prices traded on the New York Mercantile Exchange (NYMEX) and US stock prices and they find that the return volatility spillover from oil futures to stocks is very weak. Conversely however, Jones and Kaul (1996) find that US stock prices react significantly to oil shocks through the impact of these shocks on real cash flows.

Recent papers have however followed different methodological approaches, different sample data frequencies and different proxies for oil price and US stocks in order to offer more robust explanations for the nexus. Hammoudeh et al. (2004) use two US markets of oil prices namely the WTI spot and 1- to 4-month NYMEX futures prices and the proxies for the US Stocks are the

S&P oil sector stock indices which include Oil Exploration and Production, Oil & Gas Refining & Marketing, Oil-Domestic Integrated, Oil-International Integrated, and the overall Oil Composite. They employ both univariate and multivariate ARCH/GARCH models with daily data for the period July 17, 1995 to October 10, 2001. They find that there are two-way interactions between the US oil stock returns and the oil spot price and the oil futures price. Ghouri (2006) also uses WTI oil price as a proxy for oil price and the US oil stocks in petroleum products (PPP), combined petroleum products and crude oil (CPPP), crude oil alone (Crude), total oil stocks including petroleum products, crude oil and strategic petroleum reserves SPR (Total), total gasoline (TGO), total distillate (TDO) are used as proxies for US stocks. Using a simple linear regression model over the period 1995:02 to 2004:07, the paper concludes that WTI is inversely related to US oil stocks.

Kilian and Park (2009) use a structural vector auto regression (SVAR) model on monthly data covering the period 1973:01-2006:12. The aggregate US stock return is constructed from monthly returns on the Centre for Research in Security Prices (CRSP) value-weighted market portfolio while the oil price is based on US refiner's acquisition cost of crude oil, as reported by the US Department of Energy. These variables are utilized in real terms by deflating them with the US consumer price index (CPI). That is, the aggregate US real stock return was constructed by subtracting the US CPI inflation rate from the log stock returns while the real oil price was obtained by dividing the nominal price of oil by US CPI. They find that the response of aggregate US real stock returns may differ greatly depending on whether the increase in the price of crude oil is driven by demand or supply shocks in the crude oil market. They show that positive shocks to the global demand for industrial commodities cause both higher real oil prices and higher stock prices, which helps explain the resilience of the US stock market to the recent surge in the price of oil. They also find that oil demand and oil supply shocks combined account for 22 percent of the long-run variation in US real stock returns.

Following the argument of Kilian and Park (2009), Kang et al. (2014) also utilize an SVAR model to investigate how the demand and supply shocks driving the global crude oil market affect US bond market returns and they use monthly data over the period 1982:01–2011:12. While the proxy of oil price and the computation of real term are consistent with Kilian and Park (2009), the US bond return were constructed from an index of US aggregate bond holdings and the real aggregate US bond return was measured by deflating its nominal term by the US CPI. Contrary to the findings of Kilian and Park (2009), they find that a positive oil market-specific demand shock is associated with significant decreases in US bond returns. In addition, their evidence shows that the demand and supply shocks driving the global crude oil market jointly account for 30.6 percent of the long-run variation in US real bond returns.

Balcilar and Ozdemir (2012) consider monthly data from 1990:02 to 2011:07 and a Markov switching vector autoregressive (MS-VAR) model, where the causal link between the series is stochastic and governed by an unobservable Markov chain. The model allows for time-varying parameter so as to reflect changes in Granger causality over time. In their paper, US stock returns were captured by S&P500 index returns with different sub-groupings ranging from Industry, Energy, Energy Equipment & Services, to Oil & Gas & Consumable fuels, Oil & Gas Exploration & Production, Oil & Gas Storage and Transportation indexes. The oil futures price is used as a proxy for oil price. Although they do not find any lead –lag type Granger causality, the results based on the MS-VAR model clearly show that oil futures price has strong regime prediction power for a sub-grouping of S&P 500 stock index during various sub-periods in the sample, while there is a weak evidence for the regime prediction power of a sub-grouping of S&P 500 stock indexes for oil futures.

Elyasiani et al. (2012) utilize daily data from December 11, 1998 to December 29, 2006. They use NYMEX crude oil futures as a proxy for oil price and thirteen industry sectors market portfolio of NYSE, AMEX, and NASDAQ stocks as proxies for US stocks. Using the ARCH and GARCH models, they find strong evidence in support of the view that oil price fluctuations constitute a systematic asset price risk at the industry level as nine of the thirteen sectors analyzed show statistically significant relationships between oil-futures return distribution and industry excess return. These industries are affected either by oil futures returns, oil futures return volatility or both. In general, excess returns of the oil-user industries are more likely to be affected by changes in the volatility of oil returns, than those of oil return itself.

Fan and Jahan-Parvar (2012) utilize WTI spot and NYMEX light sweet crude futures prices for oil price while the US stock returns were computed from average monthly value weighted returns on forty nine US industry level portfolios of NYSE, AMEX, and NASDAQ stocks. They employ both linear regression model and vector autoregressive (VAR) model with monthly data from 1979:01 to 2009:01. They find that oil-price predictability is concentrated in relatively small number of industries.

Alsalman and Herrera (2013) estimate a simultaneous equation model that nests both symmetric and asymmetric responses of stock returns to positive and negative oil price innovations using monthly data between 1973:01 and 2009:12. The US stock returns were proxied by the excess returns on all NYSE, AMEX, and NASDAQ stocks. They find in-sample evidence that the oil price increase helps to forecast aggregate US stock returns as well as industry-level returns one-year ahead. They also demonstrate that the size of oil price shocks matters in that doubling the size of the shocks more (or less) than doubles the size of the response of stock returns. In essence, the effect of a 2.s.d innovation is just about double the magnitude of the impact of a 1.s.d innovation.

Mollick and Assefa (2013) employ the GARCH and MGARCH-DCC models using daily data from January 1999 to December 2011. They use S&P 500, Dow Jones, NASDAQ, and Russell 2000 indexes returns as proxies for US stock returns and WTI for oil price. They find that prior to the financial crisis, US stock returns are slightly (negatively) affected by oil prices and by the exchange rate (USD/Euro). However, from mid-2009 onwards, the stock returns are found to be positively affected by oil prices and a weaker USD/Euro.

Conrad et al. (2014) use a modified Dynamic Conditional Correlations- Mixed Data Sampling (DCC-MIDAS) specification proposed in Colacito et al. (2011) and further extended by Engle et al. (2013) and their data covers the period from 1993:01 to 2011:11. For the stock series, they employ the daily returns on the CRSP value-weighted portfolio, which is based on all NYSE, AMEX and NASDAQ stocks while the oil price is proxied by WTI. They find that variables that contain information on current and future economic activity are helpful predictors of changes in the oil – US stock correlation.

Table 1: Summary of literature on Oil Price-US Stocks nexus

Author(s)	**Period**	**Proxy(ies) for US Stock**	**Proxy(ies) for Oil Price**	**Model**	**Findings**
Hammoudeh et al. (2004)	July 17, 1995 to October 10, 2001	S&P oil sector stock indices	WTI spot and NYMEX future prices	Univariate and Multivariate ARCH and GARCH model	Significant bi-directional relationship
Ghouri (2006)	1995:02 to 2004:07	US Oil stocks	WTI	Simple Linear Regression	WTI is inversely related to US oil

				Model	stocks
Kilian and Park (2009)	1973:01 to 2006:12	Aggregate US stocks	Composite refiners' acquisition cost (RAC)	Structural VAR (SVAR)	There is shock spillover of about 22 percent from oil to stock
Balcilar and Ozdemir (2012)	1990:2 to 2011:07	S&P 500 index return	Oil futures price	Markov Switch VAR (MS-VAR)	Strong causality from oil to stock and weak causality from stock to oil
Elyasiani et al. (2012)	Dec. 11, 1998 to Dec. 29, 2006	NYSE and NASDAQ stocks	NYMEX crude oil futures	ARCH and GARCH model	Oil returns volatility mostly affect oil user industries
Fan and Jahan-Parvar (2012)	1979:01 to 2009:01	NYSE, AMEX and NASDAQ stocks	WTI spot and NYMEX light sweet crude future prices	OLS and VAR	Oil price predictability is concentrated in small number of industries
Alsalman and Herrera (2013)	1973:01 to 2009:12	NYSE, AMEX and NASDAQ stocks	US Composite refiners' acquisition cost (RAC)	Simultaneous Equation Model (SEM)	Oil price increase helps to forecast aggregate US stock return
Mollick and Assefa (2013)	1999:01 to 2011:12	S&P 500, Dow Jones, NASDAQ and Russel 2000 indexes	WTI	GARCH and MGARCH-DCC	Oil price affects US stock returns negatively before, and positively after financial crisis
Conrad et al. (2014)	1993:01 to 2011:11	NYSE, AMEX and NASDAQ stocks	WTI	DCC-Mixed Data Sampling (DCC-MIDAS)	Oil-stock correlation are better determined by conditional variables
Kang et al. (2014)	1982:01 to 2011:07	Aggregate US Bonds	US Composite refiners' acquisition cost (RAC)	Structural VAR (SVAR)	There is shock spillover of about 30.6 percent from oil to stock

Source: Compiled by the authors

In addition to the prominent papers on oil price and US stocks, there are several other studies focusing on either oil exporting or net oil importing countries or both. For example, Bjornland (2009) and Jimenez-Rodriguez and Sanchez (2005) argue that an oil price increase is expected to have a positive effect in an oil exporting country, as the country's income will increase. Consequently, these increases in income are expected to lead to a rise in expenditure and investments, which in turn creates greater productivity including the stock markets. Thus, a positive association is anticipated between oil price and stock market for an oil-exporting country. Similarly, Arouri et al. (2011b) cover stock markets in the Gulf Cooperation Council (GCC) countries (oil-expoters) and Arouri et al. (2012) examine stock markets in Europe (oil-importers) and both papers find similar evidence suggesting substantial returns and volatility spillovers between world oil prices and these stock markets. Recently, Jouini and Harrathi (2014) revisit the empirical evidence of the volatility interactions among the GCC stock markets and world oil price. Their findings show evidence of shock and volatility linkages among GCC stock and oil markets, and reveal that the spillover effects are more apparent for volatility patterns. They also indicate that the stock and oil markets exhibit asymmetry in the conditional variances.

For an oil-importing country however, oil price increase will drive a higher cost of production being one of the critical components of production, the burden of which is usually transferred to the final consumers, in terms of higher consumer prices. As a consequence, consumer expenditure and investment on goods and services will decline in such circumstances including demand for stocks. Thus, a negative relationship is expected between oil price and stock market for an oil-importing country (see also Filis et al., 2011 for a survey of the literature). For example, Park and Ratti (2008) having examined 13 European countries, they conclude that positive oil price shocks cause positive returns for the Norwegian stock market (oil-exporter), whereas the opposite happens to the rest of the 13 European stock markets (oil-importers). Also, Wang et al. (2013) examine both oil-importing and oil-exporting countries. Their findings can be summarized as follows: First, they find that the magnitude, duration, and even direction of response by stock market in a country to oil price shocks highly depend on whether the country is a net importer or exporter in the world oil market, and whether changes in oil price are driven by supply or aggregate demand. Second, they show that the relative contribution of each type of oil price shocks depends on the level of importance of oil to national economy, as well as the net position in oil market and the driving forces of oil price changes. Third, they demonstrate that the effects of aggregate demand uncertainty on stock markets in oil-exporting countries are much stronger and more persistent than in oil-importing countries. In conclusion, their findings reveal that positive aggregate and precautionary demand shocks are shown to result in a higher degree of co-movement among the stock markets in oil-exporting countries, but not among those in oil-importing countries.

In addition, Agren (2006) considers stock markets in five major developed countries (Japan, Norway, Sweden, the U.K., and the US); Malik and Hammoudeh (2007) cover US and Gulf equity markets and Malik and Ewing (2009) focus on US and their empirical results support the existence of significant transmission of shocks and volatility between world crude oil prices and different stock markets irrespective of the nature of the countries. Surprisingly however, Apergis and Miller (2009) find that stock markets (both from oil-importing and oil-exporting countries) tend not to react to oil price shocks (either positive or negative).

Like the studies on US, also, a vast majority of these studies have employed different methodological approaches such as vector autoregressive (VAR) model, vector error-correction model (VECM), univariate and multivariate GARCH-type models including the BEKK (Baba, Engle, Kraft and Kroner over parameterization), CCC (Constant Conditional Correlation) and DCC (Dynamic Conditional Correlation) with different country or regional case studies. For example, Agren (2006) uses an asymmetric version of the BEKK – GARCH(1,1) for stock markets in five major developed countries (Japan, Norway, Sweden, the U.K., and the US); Malik and Hammoudeh (2007) uses the same model for US and Gulf equity markets and Malik and Ewing (2009) similarly employ bivariate BEKK–GARCH(1,1) for five US sector indices and their empirical results support the existence of significant transmission of shocks and volatility between world crude oil prices and different stock markets. In a similar fashion, Jouini and Harrathi (2014) adopt the the BEKK-GARCH process to evaluate the volatility interactions among the GCC stock markets and world oil price. Also, Arouri, et al. (2011b) employ VAR(1)–GARCH(1,1) for stock markets in the Gulf Cooperation Council (GCC) countries and Arouri, et. al. (2012) employ the same model for the stock markets in Europe. Wang et al. (2013) however use structural VAR model examine the relationship between oil prices and stock Oil price shocks and stock market activities between oil-importing and oil-exporting countries.

Overall, the empirical findings from the various studies suggest that the choice of methodology, measurement of variables and the peculiar characteristics of the country (ies) under consideration may influence oil price-stock market nexus. Thus, generalizing with the findings of the existing literature may offer misleading policy inferences. Summary of methodological review for countries other than the US is presented in the table below:

Table 2: Summary of literature on Oil - Stocks nexus for countries other than US

Author(s)	**Period**	**Proxy for Stock Market**	**Proxy for Oil Market**	**Model**
Agren (2006)	Weekly data from1989:1 to 2005:17.	Stock Indices	Brent crude oil price	VAR(2)-BEKK-GARCH & VAR(2)-ABEKK-GARCH
Arouri et al. (2011b)	June 7, 2005 to February 21, 2010	Stock market indices	Brent spot price	VAR(1)-GARCH(1,1)
Malik and Hammoudeh (2007)	February 14, 1994 to December 25, 2001	Stock indices	Spot price for West Texas Intermediate (WTI)	BEKK-MGARCH
Park and Ratti (2008)	1986:1 to 2005:12	Real stock returns	Real Brent crude oil returns	VAR
Wang et al. (2013)	January 1999 to December 2011	Stock indices	West Texas Intermediate (WTI) crude oil price.	SVAR

Source: Compiled by the authors

3.0 Data and Preliminary analyses

Essentially, this study makes use of daily data for both Brent and West Texas Intermediate (WTI) crude oil price as proxies for oil price and the S&P 500 stock index as a proxy for US stock market (USST) for the period from 02/01/2002 to 04/04/2014. Consequently, a total number of 3198 observations was generated. The data for both Brent and WTI crude oil prices are obtained from Thompson Reuters through the database of Energy Information Administration (EIA) while data for S&P 500 stock index is obtained from S&P Dow Jones Indices LLC. The returns of the series (r_t) are computed as the first difference of the natural logarithm of the level series (P_t); this expressed in the equation below:

$$r_t = 100*[\Delta \log(P_t)] \quad (1)$$

Thus, positive/negative returns in oil price will represent return gain/loss in oil market while positive/negative returns in US stock index will imply investment gain/loss in US stock market.

In addition, this section examines the statistical properties of both level and returns series and also confirms relevant stylized facts about financial time series variables. In essence, we present descriptive statistics and conduct appropriate tests for serial correlation and time varying conditional variance property i.e. ARCH effects.

Table 3 below shows the descriptive statistics augmented with the results for serial correlation using Ljung-Box Q-statistics test and for ARCH effects using ARCH-LM test by Engle (1982). Also included is the result for unconditional correlation between Brent returns and US stock returns on one hand and between WTI returns and US stock returns on the other hand.

Table 3: Descriptive Statistics

Statistics	Brent		WTI		US Stock	
	P_t	r_t	P_t	r_t	P_t	r_t
Mean	71.952	0.052	69.111	0.0489	1236.670	0.014
Median	70.080	0.073	70.605	0.132	1219.825	0.062
Maximum	143.950	20.746	145.310	16.413	1890.900	10.957
Minimum	18.170	-16.832	18.020	-15.190	676.530	-9.469
Std. Dev.	32.025	2.134	27.322	2.288	234.905	1.260
Skewness	0.061	0.012	-0.003	-0.170	0.380	-0.197
Kurtosis	1.749	10.112	2.108	8.284	3.004	12.749
Jarque-Bera	210.299	6739.693	105.831	3735.700	77.207	12681.30
Probability	(0.000)*	(0.000)*	(0.000)*	(0.000)*	(0.000)*	(0.000)*
ARCH LM test (1)	220.16*	213.96*	696.29*	280.03*	104.63*	124.20*
ARCH LM test (5)	51.142*	50.448*	195.24*	126.23*	154.25*	210.47*
ARCH LM test(10)	27.273*	29.460*	100.88*	66.194*	104.02*	128.26*
LB-Q(5)	0.689	2.866	30.325*	23.841*	29.912*	14.128*
LB-Q(10)	3.366	9.616	34.160*	40.253*	45.912*	29.478*
LB-Q(20)	14.909	27.779***	50.233*	54.913*	85.321*	63.704*

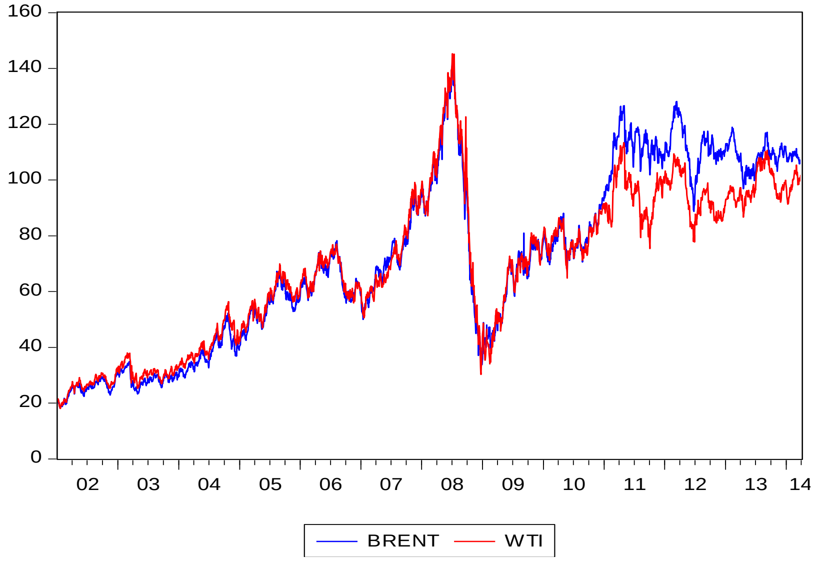
LB-Q^2(5)	278.52*	288.36*	1076.1*	976.75*	1093.6*	1435.7*
LB-Q^2(10)	343.55*	410.76*	1246.3*	1453.5*	2319.6*	2863.2*
LB-Q^2(20)	507.03*	682.77*	1565.6*	2472.3*	3818.5*	4894.4*
Unconditional Correlation (ρ_{13})	0.6751	0.1422	-	-	0.6751	0.1422
Unconditional Correlation (ρ_{23})	-	-	0.6643	0.2445	0.6643	0.2445
Observations	3198	3197	3198	3197	3198	3197

Source: Compiled by the Authors

Note: *, ** and *** imply rejection of null hypothesis for normality using JB statistic, for no ARCH effects using ARCH LM test and for no autocorrelation using Ljung-Box Q-statistic test at 1%, 5% and 10% respectively. Also, the ranking of variables for unconditional correlation assumes 1, 2 and 3 for Brent price/return, WTI price/return and US stock price/return respectively.

From table 3 above, all the level and return series show large margin between minimum and maximum values which suggests presence of large variance. Meanwhile, as pointed by the standard deviation statistic, US stock appears to be the most volatile level series followed by Brent, while WTI appears to the least volatile level series. However, the reverse hold for the return series, thus, the US stock return takes the minimum standard deviation and as such appears to be the least volatile return series. This is immediately followed by Brent return, and then by WTI return, which eventually becomes the most volatile return series. In addition, the skewness statistic shows that level and return series for Brent is positively skewed while it is negatively skewed for WTI. Whereas, for the US stock, it shows that the level series is positively skewed while the return series is negatively skewed. This implies that there is high tendency of having extreme positive values for Brent prices and return, and extreme negative values for WTI prices and return. The disparity between these two crude oil prices may be viewed as being the effect having WTI lying consistently below Brent crude price since 2011 (see Figure 1).

Figure 1: Combined graph for Brent and WTI



One the other hand, the skewness statistic reveals that the level series for the US stock is positively skewed while its return series is negatively skewed. This implies that positive extreme value in more likely to be obtained for stock price while negative extreme value is possible for stock return. Apparent distinction between skewness statistics of stock prices and its return suggests significance of inflation during the period under consideration.

Moreover, the kurtosis statistic which compares the peakedness and tailedness of the probability distribution with that of a normally distributed series shows that Brent and WTI are low peaked and thin tailed (platykurtic) at level while US stock is moderately peaked and moderately tailed (mesokurtic). However, as predicted by stylized facts on financial time series returns, the Brent, WTI and US stock returns are high peaked and fat tailed (leptokurtic). This implies that the probability that outliers may occur is higher than that of normal distribution. Meanwhile, the Jarque-Bera statistic which measures normality of the distribution using both the skewness and kurtosis statistics shows that we can reject the null hypothesis for normality for all level and return series at all conventional significant level.

We further carry out stochastic test for autocorrelation and conditional heteroscedasticity to further verify stylized facts on financial time series variables. ARCH-LM test by Engle (1982) was adopted for testing the significance of time varying conditional variance (ARCH effects) while Ljung-Box Q-statistic test was employed for testing the significance of autocorrelation or serial correlation. The results for these tests as presented in table 1 shows that we can reject the null hypothesis of no ARCH effects for all the level series and particularly for the return series at 1 percent level of significance. In addition, Q-statistic results show that there is statistically significant autocorrelation in level and return series for WTI and US stock. Whereas, level and return series for Brent are found to exhibit higher order autocorrelation, given statistically significant squared residuals.

Also, the unconditional correlation coefficients show that there is positive correlation between oil price and US stock, using either Brent or WTI as benchmark for crude oil price. This implies that there is link between oil and US stock markets and that both move in the same direction. Meanwhile, stronger unconditional correlation coefficient displayed by the level series seems to have been exaggerated due to spurious regression, while stronger correlation displayed by WTI return over Brent return may be due to the uniqueness of WTI to the US economy. Therefore, we employ quasi-maximum likelihood (QML) estimation method in this study to capture both normality and stochastic deficiencies of the series.

The second segment of this section deals with the preliminary analysis which discusses the relationship between crude oil price and the US stock market in the light of observed trend patterns in the series. Basically, we identified five trend patterns and divided the whole series into five quadrants accordingly. The analysis will be done in respect of figures 2 and 3 concurrently as similar trend pattern is observed for both WTI and Brent crude oil price.

In quadrant 1, which covered the period from January 2, 2002 to March 12, 2003 negative relationship is observed between oil price and the US stock market. There was steady rise in oil price during this period due to strike in Venezuela during 2002/2003 which reduced global oil supplies by 2.3mbd, or 3 percent of total world supply, and unrest in the oil producing region in Nigeria. In effect, as United States will have to import crude oil at higher price for both commercial and residential use, thus, we should expect rise in production cost, reduction in future cash flows of industries and eventually fall in stock prices. Report from Organization for Economic Cooperation and Development (OECD) database shows that the real total gross fixed capital formation of the United States fell by 1.8 percent in 2002, thus confirming possibility of spillover effects from oil to US stock market.

Meanwhile, in quadrant 2, which spanned from March 13, 2003 till January 23, 2007, although the increasing trend in oil price continues the US stock market appreciates to exhibit positive relationship with oil price. The rising oil price during this period may not be unconnected with increase in global demand for oil engineered by industrialization and investment boom in the emerging Asian economies. The US economy was also able to match up with the global challenge and increased demand for crude oil to serve the rising trade and investment. This suggests the possibility of spillover effects from the US stock market to oil market.

Also, in quadrant 3 which covered the period from January 24, 2007 to February 10, 2009, the relationship between oil price and the US stock market is mixed. This period recorded continuous negative stock returns in the US stock market due to failure of the sub-prime lending market which eventually led to acute economic and financial problem in the United States. Meanwhile, in the inception of this problem oil price was increasing partly due to increasing demand for oil by the emerging Asian economies particularly China and India which were able to accommodate investment outflow from the US economy. However, oil price fell in the later period of the financial crisis which also suggests possibility of spillover effects from US stock market to oil price.

Figure 2: Combined graph for WTI crude oil price and the US stock market

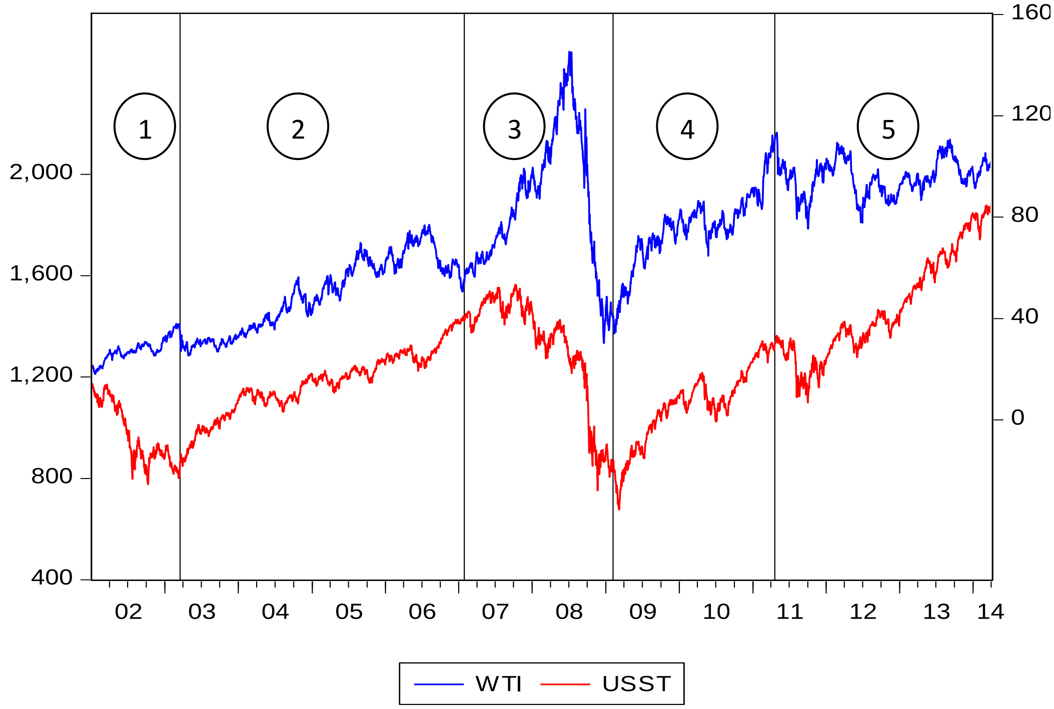
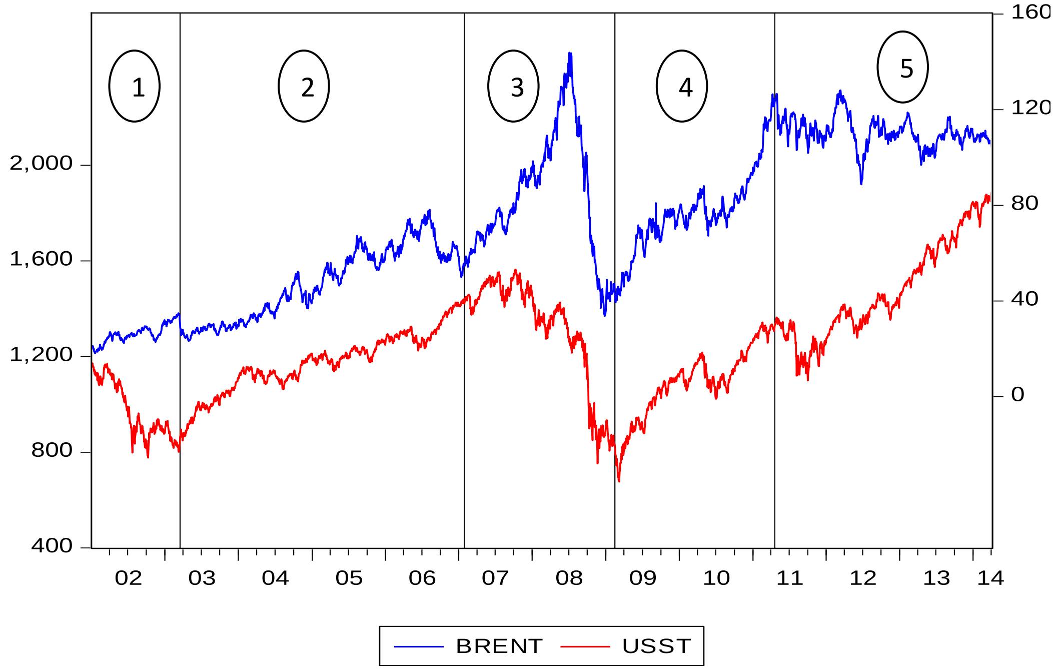


Figure 3: Combined graph for Brent crude oil price and the US stock market



In quadrant 4 which covered the period from February 11, 2009 to April 15, 2011, positive relationship is evident. Oil price further shows an increasing trend partly due to shortage supply occasioned by turmoil in oil producing North African countries and particularly political crisis in Libya. In addition, the US government set out some economic recovery programmes during the period which manifested in steady rise in investments and stock prices during this period. Increase in demand for crude oil the United States to support economic recovery may also fuel oil price increase. This also lends support to the possibility of spillover effect from the US stock market to oil market.

Lastly, in quadrant 5 which spanned from April 16, 2011 to April 4, 2014 the relationship is rather unclear. There is no definite trend pattern in oil price, but obviously, it is still on the high side. But on the other hand, the US economy still keep fit on the track of economic recovery, thus making US stock market to remain strong and healthy, and appreciates steadily to regain and rise above its position before the financial crisis. We may also conclude here that rising US stock market makes oil price to remain on the high side, thus confirming possibility of spillover effect from US stock market to oil market.

Meanwhile, the visibility of clear-cut categorization in the co-movement among the three series as discussed under five quadrants above suggest the presence of structural break. Perron (2006) suggests a suitable framework for determining structural break endogenously from the dataset and this framework is adopted in this study to test for the stationarity as well as determine the significance of structural break in the return series. The generalized test regression can be expressed as:

$$y_t = \mu + \theta DU_t + \beta t + \gamma DT_t^* + \delta D(T_1)_t + \alpha y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + e_t \; ; \; e_t \sim i.i.d. \; (0, \sigma_e^2) \tag{2}$$

where $DU_t = 1$; $DT_t^* = t - T_1$ if $t > T_1$ and 0 otherwise; $D(T_1)_t = 1$ if $t = T_1 + 1$ and 0 otherwise. The test considered is the minimal value of the t-statistic for testing that $\alpha = 1$ versus the alternative hypothesis that $|\alpha| < 1$ over all possible break dates in some pre-specified range for the break fraction $[\epsilon, 1-\epsilon]$. The implementation of the test regression follows the Innovational Outlier (IO) framework as it allows the change to the new trend function to be gradual rather than being instantaneous as assumed by the Additive Outlier (AO) framework. The results are presented in table 4 below:

Table 4: Result for Unit Root with Structural Break

Variables	Break Dates	T-stat
Brent	04/12/2008	-58.7627
WTI	19/09/2008	-58.6879
US Stock	19/11/2008	-62.3663

Note: Critical values are -5.28 and -4.62 for 1% and 5% levels of significance respectively.

Evidence from table 4 above shows that the null hypothesis of unit root could be rejected, thus suggesting that the returns for the three variables are stationary. Also, allowing for one structural break in the unit root equation, the break dates for the three variables fall within the last quarter of 2008 which coincides with the period of the global economic and financial crisis. This further reinforces the observed trends in quadrant 3 of figures 2 and 3 that reveal structural changes in the series in 2008. Thus, due to the significance of the structural break in the return series, we modify the mean equation of our proposed multivariate GARCH model by including dummy variables to capture the identified break dates. In order to validate this framework, we estimate and compare our results with other variants such as the Constant Conditional Correlation (CCC) and Dynamic Conditional Correlation (DCC) variants of VARMA-AGARCH. Details of the estimation procedure are reported in the next section.

4.0 The Model

As previously noted, this study employs the VARMA-AGARCH model by McAleer et al. (2009) which is an augmented version of VARMA-GARCH model by Ling and McAleer (2003). Thus, this model specifies conditional mean equation with vector autoregressive moving average (VARMA) and conditional variance equation with multivariate GARCH process as proposed by Ling and McAleer (2003). However, in addition to this, it recognizes the possibility of asymmetric impacts of shocks on the conditional variance rather than assuming identical asymmetric effect for equal magnitude of positive and negative shocks as suggested by VARMA-GARCH model. Eventually, VARMA-AGARCH model has become a prominent instrument for modelling interdependencies among financial time series variables in recent times when asymmetric effect of shocks has become a distinguished feature of financial markets.

Meanwhile, this study specifies and estimates VARMA(1,1)-BEKK-AGARCH(1,1) model as against the CCC and DCC variants of McAleer et al. (2009). The rationale behind this is to measure cross market asymmetric effect which is structurally not revealed by the CCC and DCC. Nonetheless, we estimate the three models in order to judge the robustness of our results.

We specify a bivariate VARMA(1,1)-BEKK-AGARCH(1,1) model under separate headings for the conditional mean equation and conditional variance equation as follows:

The Conditional Mean Equation

$$R_t = \Phi + \Psi R_{t-1} + \Theta B_t + \varepsilon_t + \Upsilon \varepsilon_{t-1} \tag{3}$$

$$\varepsilon_t = D_t \eta_t \tag{4}$$

where

- $R_t = (r_t^{usst}, r_t^{oil})'$ with r_t^{usst} and r_t^{oil} being the returns on US stock market and oil market at time *t* respectively;
- Ψ is a (2x2) matrix of coefficients of the form $\Psi = \begin{pmatrix} \psi_{11} \, \psi_{12} \\ \psi_{21} \, \psi_{22} \end{pmatrix}$;
- Φ is a (2x1) vector of constant terms of the form $(\phi^{usst}, \phi^{oil})'$;
- Θ is a (2x2) vector of coefficients for the structural break dummies $\begin{pmatrix} \varphi_{11} & \varphi_{12} \\ \varphi_{21} & \varphi_{22} \end{pmatrix}$ and $B_t = \left(b_t^{usst}, \quad b_t^{oil}\right)'$ where $b_t = 1$ if $t \geq \text{Break Date}$;
- $\varepsilon_t = (\varepsilon_t^{usst}, \varepsilon_t^{oil})'$ with ε_t^{usst} and ε_t^{oil} being error terms from the mean equations of the US stock market and oil market returns respectively;
- Υ is a (2X2) matrix of the coefficients of lagged terms of residuals in the form $\Upsilon = \begin{pmatrix} \gamma_{11} \, \gamma_{12} \\ \gamma_{21} \, \gamma_{22} \end{pmatrix}$; which explains shock spillovers between oil and US stock returns.
- $\eta_t = (\eta_t^{usst}, \eta_t^{oil})'$ refers to a (2x1) vector of independently and identically distributed errors;
- and $D_t = diag(\sqrt{h_t^{usst}}, \sqrt{h_t^{oil}})$ with h_t^{usst} and h_t^{oil} being the conditional variances of r_t^{usst} and r_t^{oil} respectively.

The Conditional Variance Equation

$$H_t = \Omega'\Omega + A'\varepsilon_{t-1}\varepsilon_{t-1}'A + C'I_{t-1}\varepsilon_{t-1}\varepsilon_{t-1}'C + B'H_{t-1}B \tag{5}$$

where A, B and C are square matrices and Ω is a lower triangular matrix defines as:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, \quad C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}, \quad \Omega = \begin{bmatrix} \omega_{11} & 0 \\ \omega_{21} & \omega_{22} \end{bmatrix}$$

From equation 5 above, H_t is the conditional variance- covariance matrix which defines market volatility. Elements of matrix A are the coefficients of ARCH term which shows the effect of shock in the own market and shock spillover from other market on the conditional volatility of a choice market. Also, elements of matrix B are the coefficients of GARCH term which shows the effect of past volatility in the own market and past volatility spillover from the other market on the conditional volatility of a choice market. It must however be noted that ARCH terms represent short term persistence volatility since the effect of shock on conditional volatility is not expected to last long. Whereas, GARCH terms represent long term persistence volatility given the autoregressive nature of conditional volatility. Also, summation of ARCH and GARCH terms for a particular market is expected to be positive and less than unity to satisfy the mean reverting condition, or in other words, for long run equilibrium in conditional volatility to be established. In addition, the magnitude of summation of these GARCH terms for a particular market determines the speed of convergence of the conditional volatility in such market to its long run equilibrium. In other words, it determines the time frame at which financial market return in question will be free from ARCH effects. The lower the summation of both terms, the longer the period of convergence to long run equilibrium.

Furthermore, elements of matrix C represent asymmetric effect coefficients with the diagonal elements showing significance of asymmetric effect for own markets while the off-diagonal elements show the significance of asymmetric effect spillover between markets in the system. The significance of asymmetric effect in a market implies that negative shock (bad news) commands higher conditional variance than positive shock (good news). In other words, negative and positive shocks do not have identical effect on the conditional variance. This effect is captured by $I_t = diag(I_t^{usst}, I_t^{oil})$ defined as a function of independently and identically distributed error term, given as:

$$I\left(\eta_t^i\right) = \begin{cases} 0, & \varepsilon_t^i > 0 \\ 1, & \varepsilon_t^i \leq 0 \end{cases} \qquad \text{for } i \text{ selected market at time t.} \tag{6}$$

It must however be noted that if matrix C is a null matrix then BEKK-VARMA-AGARCH reduces to BEKK-VARMA-GARCH. The structural and statistical properties of the model, including necessary and sufficient conditions for stationarity and ergodicity of VARMA-GARCH and VARMA-AGARCH are explained in detail in Ling and McAleer (2003) and McAleer et al. (2009) respectively.

Similarly, we specify the conditional variance for the $\text{VARMA(1,1)-CCC-AMGARCH}(1,1)$ as follows: [3]

Let $H_t^{1/2} = D_t$; then, $\text{var}\left(\varepsilon_t \middle| \Sigma_{t-1}\right) = H_t = D_t \Gamma D_t$;

where Σ_{t-1} is the past information available at time t-1, $\Gamma = E\left(v_t v_t' \middle| \Sigma_{t-1}\right) = E\left(v_t v'\right) = \left\{\rho_{ij}\right\}$ and it denotes the constant conditional correlation matrix of the unconditional shocks which is also equivalent to the constant conditional covariance matrix of the conditional shocks $\left(\varepsilon_t\right)$. And since the H_t is the conditional covariance matrix, it can be used to accommodate the

[3] Note that the distinctions among the BEKK, CCC and DCC lie in the way the variance equations are modeled. In essence, their mean equations are the same.

interdependencies of between the two assets. Following, the Ling and McAleer (2003), then H_t can be specified as:

$$H_t = \Omega + A\varepsilon_{t-1}^2 + CI_{t-1}\varepsilon_{t-1}^2 + BH_{t-1} \tag{7}$$

where $H_t = \left(h_t^{REXR}, h_t^{RASI}\right)'$ and consequently, $D_t = diag\left(h_1^{1/2}, \quad h_2^{1/2}\right)$ $\varepsilon_t^2 = \left(\varepsilon_{REXR,t}^2, \quad \varepsilon_{RASI,t}^2\right)'$, and Ω, A, and B are (2×2) matrices of constants, ARCH effects and GARCH effects respectively. $I_t = diag\left(I_t^{REXR}, \quad I_t^{RASI}\right)$ is the asymmetric effects such that $I_t = 0$ if $\varepsilon_{it} > 0$ and $I_t = 1$ otherwise.

For the conditional variance in the case of DCC-VARMA-AMGARCH model,

$$\Gamma_t = \left\{\left(diag\left(H_t\right)^{-1/2}\right)\right\} H_t \left\{\left(diag\left(H_t\right)^{-1/2}\right)\right\} \tag{8}$$

And H_t is a positive definite matrix given as:

$$H_t = \left(1-\delta_1-\delta_2\right)\bar{H} + \delta_1 I_{t-1} v_{t-1} v_{t-1}' + \delta_2 H_{t-1} \tag{9}$$

where δ_1 and δ_2 are scalar parameters to deal with the effects of previous shocks and previous dynamic conditional correlations on the current dynamic conditional correlation, and δ_1 and δ_2 are non-negative scalar parameters. By imposing the restriction $\delta_1 = \delta_2 = 0$, $\bar{H}$ reduces to the CCC (Caproin and McAleer, 2010 provide the estimation procedure for the DCC).

The goodness of fit of the models is determined with the minimum values of Akaike Information Criterion (AIC) and Schwartz Bayesian Criterion (SBC). Also, the Ljung-Box statistic is used to test for autocorrelation and the null hypothesis is that there is no autocorrelation. The McLeod-Li statistics is employed to test for ARCH effects and the underlying null hypothesis is that there are no ARCH effects in the model. For a robust model, we do not expect to reject the null hypothesis of both the Ljung-Box and McLeod-Li statistics.

5.0 Discussion of Results

Table 5 below shows the results of the BEKK variant in the presence of structural breaks. The CCC and DCC variants including the BEKK are presented in the appendix. Using the model selection criteria, we find that the BEKK variant with structural breaks outperforms all the other models. The results in table 5 are divided into two segments: the first segment presents the mean equation which analyses the return and shock spillovers between the two markets, while the second segment presents the variance equation which discusses the volatility, shock and asymmetric spillovers.

It is noteworthy that a cursory look at the results for both USST (US stocks) and Brent and USST and WTI suggests that there is no significant difference (both in terms of sign, magnitude and significance) in their estimates particularly over the full sample period. Therefore, the reaction of US stock market to movements in the two oil prices appears similar for the full sample data. Under the full sample period, starting with the mean equation, the result shows that the US stock market return in the current time does not depend on the immediate past return in the oil market. This implies that there is no immediate return spillover from oil market to US stock market. This is explained by the insignificant coefficient of ψ_{12}. Meanwhile, the result further reveals that oil market return in the current period is significantly influenced by past stock market returns. This implies that there is significant positive return spillover from the US stock market to oil market as explained by positive significant coefficient of ψ_{21}. In other words, higher stock market returns are more likely to drive higher returns in oil market and the reverse does not hold. This is not however surprising since US imports about 60 percent of its total oil consumption; thus, a sustained appreciation in stock market will typically imply an healthy US economy and a relative tranquillity in the global oil investment which may enhance oil market returns. Instances of how a sustained appreciation of stock market might have enhanced oil market returns are evident in quadrants 2 and 4 of figures 2 and 3. Apparently, the periods covered by these two quadrants coupled with some parts of quadrant 3 with similar features overwhelm others with an opposite direction. This further lends support to the overriding evidence of a positive link between oil price and US stock returns. Important lesson from this relationship is that, there is high risk tendency in oil market when the US stock market shrinks. Appropriate hedging approach may be required to mitigate such risk; this is discussed in the next section.

Furthermore, the moving average part of the mean equation shows that there is a negative bi-directional shock spillover between the two markets as depicted by the negative significant coefficients of γ_{12} and γ_{21}. This implies that unanticipated occurrence in the oil market affects US stock market return negatively. In the same vein, oil market returns are sensitive to unanticipated shocks in the US stock market. This result is partly supported by Kilian and Park (2009) and Kang et al. (2014), with both confirming shock spillover from oil to US stock market. Meanwhile, period dummies show that there is no significant structural change in US Oil and stock market returns before and after the structural break. Nonetheless, we also run separate regressions using sub-samples for pre- and post- break periods. The main distinction noticed in the mean equation is that the return spillover effect from stock to oil market tends to disappear after the structural break as shown by insignificant coefficient of ψ_{21} in the "USST & WTI" model. This may however be due to serious advancement in Shale technology after 2010 by the US which appears to have reduced the price of WTI relative to Brent (see Figure1), thus weakening the link between oil and US stock markets.

Let us now turn to the conditional variance equation. First, it is worth emphasizing that a direct interpretation of the estimated coefficients is difficult due to the quadratic form of the conditional variance and covariance matrix in the BEKK model (Schmitz and von Ledebur, 2011). Typical elements of the resulting variance and covariance equations for the estimates of bivariate VARMA-BEKK-AGARCH can be expressed as:

$$\begin{aligned} h_{11,t} &= \omega_{11}^2 + a_{11}^2\varepsilon_{1,t-1}^2 + a_{21}^2\varepsilon_{2,t-1}^2 + 2a_{11}a_{21}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + c_{11}^2\varepsilon_{1,t-1}^2 I_{1,t-1} + c_{21}^2\varepsilon_{2,t-1}^2 I_{1,t-1} \\ &\quad + 2c_{11}c_{21}\varepsilon_{1,t-1}\varepsilon_{2,t-1} I_{1,t-1} + b_{11}^2 h_{11,t-1} + b_{21}^2 h_{22,t-1} + 2b_{11}b_{21}h_{21,t-1} \end{aligned} \tag{10}$$

$$\begin{aligned} h_{21,t} &= \omega_{21}\omega_{22} + a_{11}a_{22}\varepsilon_{1,t-1}^2 + a_{21}a_{22}\varepsilon_{2,t-1}^2 + \left(a_{21}a_{12} + a_{11}a_{22}\right)\varepsilon_{1,t-1}\varepsilon_{2,t-1} + c_{11}c_{22}\varepsilon_{1,t-1}^2 I_{1,t-1} + c_{21}c_{22}\varepsilon_{2,t-1}^2 I_{1,t-1} \\ &\quad + \left(c_{21}c_{12} + c_{11}c_{22}\right)\varepsilon_{1,t-1}\varepsilon_{2,t-1} I_{1,t-1} + b_{11}b_{22}h_{11,t-1} + b_{21}b_{22}h_{22,t-1} + \left(b_{21}b_{12} + b_{11}b_{22}\right)h_{12,t-1} \end{aligned} \tag{11}$$

$$\begin{aligned} h_{22,t} &= \omega_{22}^2 + a_{12}^2\varepsilon_{1,t-1}^2 + a_{22}^2\varepsilon_{2,t-1}^2 + 2a_{12}a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + c_{12}^2\varepsilon_{1,t-1}^2 I_{1,t-1} + c_{22}^2\varepsilon_{2,t-1}^2 I_{1,t-1} \\ &\quad + 2c_{12}c_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} I_{1,t-1} + b_{12}^2 h_{11,t-1} + b_{22}^2 h_{22,t-1} + 2b_{12}b_{22}h_{21,t-1} \end{aligned} \tag{12}$$

Essentially, equations (10) and (12) measure own volatility (i.e. conditional variance equations) for US stock market and oil market respectively while equation (11) measures volatility spillover between the two markets (i.e. conditional covariance equation). For the different sample data, the model is stationary since $\left(a_{11}^2 + b_{11}^2\right) < 1$ and $\left(a_{22}^2 + b_{22}^2\right) < 1$ and are both statistically significant. However, all the sums are close to one indicating that the variance process for each period reverts slowly to the mean. Thus, volatility in both the stock market and oil market exhibits weak mean reversion.

Under the full sample period, with the exception of b_{12}, b_{21} c_{12} and c_{21}; most of the parameters in equation (10) are statistically significant and positive (based on calculations from table 5). Therefore, volatility of US stock returns is significantly characterized by its own lagged conditional variance $\left(b_{11}^2\right)$, lagged own shocks $\left(a_{11}^2\right)$, own asymmetric shocks $\left(c_{11}^2\right)$ and shocks of the oil market (whether Brent or WTI) $\left(a_{21}^2\right)$. However, we find no evidence for long term volatility persistence from oil market to the US stock market and no cross-market shock spillovers since b_{21}^2, c_{21}^2, $c_{11}c_{21}$, and $b_{11}b_{21}$ are not statistically different from zero. This finding has far reaching implications. First, unanticipated events in the US stock market are capable of fuelling high volatility in the market. Second, negative shocks in the US stock market have greater impact on the volatility of the market than positive shocks of the same magnitude. Third, volatility of the market in one period has the potentiality of driving higher volatility in the immediate succeeding period. And fourth, shocks in the oil market can increase US stock market volatility. This evidence further validates the comments by financial analysts/market analysts with headlines such as "*Oil Spike Pummels Stock Market*" (Wall Street Journal) and "*US Stocks Rally as Oil Prices Fall*" (Financial Times).

Like the US stock market, the conditional variance for the oil market is also characterized by its own lagged conditional variance $\left(b_{22}^2\right)$, lagged own shocks $\left(a_{22}^2\right)$, own asymmetric shocks $\left(c_{22}^2\right)$ and shocks of the US stock market $\left(a_{12}^2\right)$. The underlying intuition behind this evidence is not different from the conditional variance of US stock market. In essence, unanticipated events in the oil market can stimulate higher volatility in the market. Also, there is evidence of significant own asymmetric effects implying that "bad news" tends to increase volatility more than "good news" in the oil market. In addition, shocks in the US stock market can raise the level of oil market volatility although the asymmetric nature of these shocks is weak. This is not unexpected; for example, improvements in stock market may imply buoyant economic activities (including production) which may lead to an increased demand for oil with attendant implications on oil price volatility. Conversely too, a slowdown in the economy may signify lower production which may consequently translate into a decrease in the demand for oil. By implication, this can create disruptions or uncertainty about future supply of oil which can lead to higher volatility in prices. Relating these findings with those of the conditional variance for the US stock market suggests bidirectional shock spillovers between the two markets. In fact, the incessant declines in the oil price in recent times have been partly attributable to the global economic slowdown including the stock market.

With regard to the sub-samples, the evidence of own asymmetric shock effects and bidirectional shock spillovers is still sustained for both pre- and post- break periods. In addition, during the pre-break period, cross market asymmetric shock spillover and volatility spillover from oil market to the US market were noticed for WTI (since c_{21} and c_{11} are statistically significant and positive only in USST &WTI results). Therefore, high volatility and asymmetric information shocks of WTI market accentuated US stock market volatility more than the Brent market during the pre-break period. However, the impact of volatility spillover from oil market to stock market became pronounced for both WTI and Brent during the post-break period given the significance of b_{21}^2 and $(b_{11}b_{21})$. Thus, the global economic meltdown heightened the transmission of oil market volatility into stock market volatility in the US and in fact also strengthened the Brent market in the US.

Our diagnostic tests show robustness of our results based on the Ljung-Box and McLeod-Li statistics. The results of the Ljung-Box test indicate that we cannot reject the null of no serial correlation likewise the McLeod-Li statistics support the adequacy of the ARCH and GARCH terms in the model.

Table 5: Estimation results for VARMA- BEKK-AGARCH model

	FULL SAMPLE		BEFORE BREAK		AFTER BREAK	
Mean Equation	USST & Brent	USST & WTI	USST & Brent	USST & WTI	USST & Brent	USST & WTI
ϕ_{10}	-0.007(0.013)	-0.005(0.014)	0.002(0.017)	0.006(0.017)	0.041(0.022)*	0.058(0.021)*
ψ_{11}	-0.061(0.018)*	-0.052(0.017)*	-0.079(0.025)*	-0.075(0.023)*	-0.042(0.027)	-0.051(0.027)***
ψ_{12}	0.002(0.007)	-0.88e-03(0.007)	-0.004(0.009)	-0.009(0.008)	0.004(0.013)	0.020(0.014)
γ_{11}	0.71e-03(0.000)*	-0.011(0.000)*	0.035(0.000)*	0.002(0.000)*	-0.477 (0.000)*	-0.016(0.000)*
γ_{12}	-0.002(0.000)*	0.015(0.000)*	-0.031(0.000)*	-0.006(0.000)*	0.041(0.000)*	-0.006(0.000)*
φ_{11}	-1.042(0.841)	0.135(0.464)				
φ_{12}	1.101(0.843)	-0.071(0.463)	-	-	-	-
ϕ_{20}	0.070(0.013)	0.065(0.043)	0.077(0.049)	0.117(0.051)**	0.018(0.035)	0.023(0.038)
ψ_{21}	0.156(0.024)*	0.104(0.030)*	0.139(0.039)*	0.150(0.044)*	0.221(0.038)*	0.027(0.045)
ψ_{22}	-0.029(0.000)*	-0.035(0.016)**	-0.001(0.025)	-0.055(0.024)**	-0.067(0.025)*	0.015(0.025)
γ_{21}	-0.006(0.000)*	0.18e-04(0.000)*	-0.008(0.000)*	-0.009(0.000)*	-0.013(0.000)*	-0.123(0.000)*
γ_{22}	-0.007(0.000)*	-0.007(0.000)*	-0.002(0.000)*	0.014(0.000)*	-0.011(0.000)*	-0.003(0.000)*
φ_{22}	0.445(0.037)*	-0.326(0.674)	-	-	-	-
φ_{21}	-0.489(0.006)*	0.279(0.674)	-	-	-	-
Variance Equation						
ω_{11}	0.111(0.011)*	-0.105(0.011)*	0.096(0.017)*	0.056(0.011)*	0.151(0.017)*	0.137(0.018)*
ω_{21}	-0.011(0.025)	-0.072(0.033)**	-0.183(0.046)*	0.402(0.100)*	-0.067(0.035)***	-0.027(0.047)
ω_{22}	0.104(0.027)*	0.141(0.030)*	0.364(0.005)*	0.750(0.020)*	0.098(0.048)**	0.152(0.052)*
a_{11}	0.084(0.019)*	0.063(0.024)*	0.008(0.039)	-0.014(0.025)	0.224(0.026)*	0.248(0.027)*
a_{12}	-0.116(0.018)*	-0.191(0.026)*	-0.097(0.041)**	0.191(0.091)**	-0.109(0.035)*	0.185(0.047)*
a_{21}	-0.031(0.007)*	-0.032(0.008)*	0.018(0.017)	-0.007(0.009)	-0.041(0.016)**	-0.065(0.017)*

a_{22}	0.160(0.017)*		0.141(0.021)*		-0.024(0.075)		0.252(0.048)*		0.234(0.016)*		0.181(0.036)*	
b_{11}	0.955(0.004)*		0.958(0.004)*		0.967(0.004)*		0.968(0.003)*		0.923(0.009)*		0.910(0.010)*	
b_{12}	0.010(0.005)		0.007(0.007)		-0.006(0.010)		0.024(0.016)		0.012(0.012)		-0.036(0.023)	
b_{21}	0.003(0.002)		-0.13e-03(0.002)		0.003(0.002)		-0.015(0.002)*		0.014(0.006)**		0.023(0.006)*	
b_{22}	0.975(0.003)*		0.972(0.004)*		0.964(0.001)*		0.855(0.012)*		0.963(0.005)*		0.964(0.010)*	
c_{11}	0.356(0.019)*		0.353(0.018)*		0.332(0.025)*		0.322(0.017)*		0.350(0.039)*		0.353(0.043)*	
c_{12}	-0.011(0.031)		0.011(0.036)		0.030(0.048)		-0.080(0.093)		0.072(0.050)		-0.075(0.075)	
c_{21}	0.69e-05(0.010)		0.002(0.036)		-0.011(0.011)		0.015(0.008)***		0.029(0.023)		0.022(0.024)	
c_{22}	0.195(0.022)*		0.217(0.008)*		0.238(0.022)*		0.289(0.056)*		0.167(0.051)*		0.241(0.049)*	
Model Diagnostics												
AIC	6.799		6.834		6.970		7.029		6.598		6.572	
SBC	6.854		6.889		7.047		7.105		6.692		6.666	
Log-L	-10835.9		-10891.52		-6224.04		-6276.08		-4597.02		-4578.95	
Obs.	3196		3196		1793		1793		1401		1401	
Residual Diagnostics for Independent Series												
	USST	Brent	USST	WTI	USST	Brent	USST	WTI	USST	Brent	USST	WTI
Ljung-Box (20)	14.335	10.267	14.033	17.264	16.964	12.413	16.579	19.596	15.196	9.585	15.712	20.929
Ljung-Box (40)	30.815	33.910	30.884	50.080	31.864	41.976	32.102	41.181	43.906	37.458	46.974	48.173
McLeod-Li (20)	32.009**	22.061	30.307	22.396	20.805	23.851	21.575	21.126	40.110*	12.953	37.008**	26.344
McLeod-Li (40)	43.147	32.588	40.940	36.234	32.115	41.985	33.420	47.154	56.898**	20.515	52.217***	51.554

Source: Compiled by the Authors

Note: Parameters in mean and variance equations are as defined in the model. The US stock is ranked (1) across the models. Brent is ranked (2) in the case of US stock – Brent nexus while WTI is ranked (2) in the case of US stock – WTI nexus. Also, *, ** and *** represent level of significance at 1%, 5% and 10% respectively. Best model is selected based on minimum values of Akaike Information Criterion (AIC) and Schwartz Bayesian Criterion (SBC). Note that AIC and SBC are not comparable for different samples.

6.0 Portfolio management between oil price and US stocks

The significance of volatility spillovers between two markets implies that investors' assets in both markets are volatile and susceptible to risk and uncertainty. One of the most prominent methods of avoiding risk in volatile assets is hedging. Hedging is done by engaging in futures contract and it has advantage of minimizing unwanted risk without reducing expected returns. This study analyzed two important hedging strategies necessary for effective portfolio management between oil and US stock assets. Thus, we construct optimal portfolio weight (OPW) to determine the optimal amount of each asset to be included in the investment portfolio and optimal hedge ratio (OHR) to determine the rate at which long position of one dollar in one market could be hedge by taking short position in the other market, such that risk is minimized without reducing returns. According to Kroner and Ng (1998), optimal weight of holding two assets - Oil (o) and US stock (s), is given by:

$$w_{os,t} = \frac{h_t^s - h_t^{os}}{h_t^o - 2h_t^{os} + h_t^s} \tag{13}$$

and,

$$w_{os,t} = \begin{cases} 0, & if \;\; w_{os,t} < 0 \\ w_{os,t} & if \;\; 0 \le w_{os,t} \le 1 \\ 1, & if \;\; w_{os} > 1 \end{cases} \tag{14}$$

where $w_{os,t}$ refers to the weight of oil in one-dollar of two assets portfolio at time t, h_t^{os} is the conditional covariance between oil and US stock returns at time t, and the optimal weight of US stock in one-dollar of two assets portfolio will be $1-w_{os,t}$. In addition to evaluating the optimal portfolio management, we also investigate how sensitive is the results to structural break. For instance, the average optimal weight of oil in one-dollar oil-US stock portfolio for the full sample with structural break is higher than the one obtained without structural break. Specifically, the optimal weight of WTI in one-dollar WTI-US stock portfolio is 16.54 percent with structural break and 16.12 percent without break. Also, the optimal weight of Brent in one-dollar Brent-US stock portfolio is 20.75 percent with break and 20.18 percent without break.

Meanwhile, when we consider sub-samples using pre- and post- break periods, we find that the results differ markedly from the full sample. Thus, the optimal portfolio ratio obtained for the full sample may appear to be the best performer over the entire period; however, this may not be valid for sub-samples particularly when there is evidence of structural break. The optimal weight is 66.49 percent for Brent and 49.49 percent for WTI before the structural break. However, during the post-break period, the optimal weight is 22.51 percent for WTI and 22.66 percent for Brent. In effect, one dollar investment in oil-US stock portfolio is optimally combined at different values for both oil and US stocks based on the data used. In terms of interpretation, for the full sample with structural break, one dollar investment in oil-US stock portfolio is optimally combined at 16.54 percent for oil (using WTI) and 83.56 percent for US stocks. The same interpretation can be extended to the sub-samples.

Table 6a: Optimal portfolio weight and hedge ratio

Optimal ratio	Full Sample with Structural Break		Full Sample without Structural Break	
	Brent	WTI	Brent	WTI
$w_{os,t}$	0.2075	0.1654	0.2018	0.1612
$\beta_{os,t}$	0.0958	0.1458	0.0820	0.1220

Source: Computed by the authors

Table 6b: Optimal portfolio weight and hedge ratio

Optimal ratio	Before Break		After Break	
	Brent	WTI	Brent	WTI
$w_{os,t}$	0.6649	0.4949	0.22660	0.2251
$\beta_{os,t}$	0.3413	0.2741	-0.13175	0.1181

Source: Computed by the authors

On the other hand, optimal hedge ratio is defined according to Kroner and Sultan (1993) which considered a portfolio of two assets and submit that the risk of the investment portfolio is minimized if a long position of one dollar in oil market can be hedged by a short position of β_t dollars in U.S stock market (see also Arouri et al. 2011b). The formulation for the optimal hedge ratio between oil and US stocks is given by:

$$\beta_{os,t} = \frac{h_t^{os}}{h_t^{s}} \tag{15}$$

In a similar fashion as reported in tables 6a and 6b, the behaviour of the optimal hedge ratio is not different from the optimal weight ratio. That is, hedging effectiveness may be exaggerated when the presence of significant structural break is ignored. For example, it is approximately 9.58 percent for Brent and 14.58 percent for WTI over the full sample with structural break while it is 8.80 percent and 12.20 percent for Brent and WTI respectively for the full sample without structural break. Clearly, the average optimal hedging ratio for the full sample when structural break is considered is higher than the ratio obtained without structural break whether WTI or Brent is used as a proxy for oil market.

Also, optimal hedge ratio over the full sample may not be the best performer for the sub-samples. As shown in table 6b, optimal hedge ratio is 34.13 percent for Brent-US stocks and 27.14 percent for WTI-US stocks before the structural break. However, during the post-break period, it is computed as -13.18 percent for Brent and 11.81 percent for WTI. This is an indication that WTI-US stock portfolio was more optimal than Brent-US stock portfolio before the global economic downturn. However, after the crisis, Brent took over. This evidence is also supported by the results of the full sample with structural break. As shown in table 6a, one dollar long position in oil market (proxied by WTI) can be hedged by shorting 15 cents of US stocks and 10 cents if Brent is considered. A cursory look at the combined graph for WTI and Brent (figure 1) depicts that in recent times, WTI has continued to trade lower than Brent. Therefore, on average, it may be more optimal to hold Brent-US Stock portfolio than WTI-US stock portfolio.

This submission may lend support to the report of Exchange-Traded Fund (ETF) in February 22, 2011 that states that "... *a look at the recent history of the Oil & Gas ETFdb Category reveals another performance gap that has an intriguing explanation; recently, the United States Oil Fund (USO) was down about 1% on the year, while the United States Brent Oil Fund (BNO) was up about 13%. In other words, the performance delta between these two products over the last seven weeks or so was about 15%–a considerable gap considering that the underlying holdings have historically exhibited a very strong correlation."* Also in May 29, 2012, it reports that *"... Since its inception in mid 2010, BNO and Brent oil alike have put a beating on WTI with a return around 75% for the fund. In comparison, USO has gained 25% – which isn't bad."*

7.0 Concluding remarks

In this paper, we examine possible interactions between oil and US stock markets using the VARMA-BEKK-AGARCH model which allows for spillover analysis as well as own and cross market asymmetric effects. Thereafter, we modify the model to account for statistically significant structural breaks and then we compare our results with other variants of multivariate GARCH models such as the CCC and DCC variants. Based on standard model selection criteria, the BEKK variant outperforms all the other variants. On this basis, we compute the optimal weight and hedge ratios under different sample data.

First, we find structural break dates that coincide with the period of global financial crisis for all the variables of interest. Drawing from the full sample results, we establish empirical evidence in support of a statistically significant positive return spillover from US stock market to oil market and bidirectional shock spillover between the two markets. In addition, there is significant own asymmetric shocks in both markets; but there is no evidence of significant cross-market asymmetric effects. On the basis of the sub-samples, we find that cross market asymmetric shock spillover and volatility spillover from oil market to the US market were only significant for WTI during the pre-break period. This seems to suggest that increasing volatility and the presence of asymmetric information shocks in WTI market accentuated US stock market volatility more than the Brent market during the pre-break period. However, after the global economic meltdown, the transmission of oil market volatility (for both WTI and Brent) into stock market volatility in the US heightened.

With respect to portfolio management, using the full sample with structural break, we find that the optimal weight of oil in oil-US stock portfolio is 22.51 percent and one dollar long position in oil market (proxied by WTI) can be hedged by shorting 15 cents of US stocks and 10 cents if Brent is considered. However, these results differ from the full sample without structural break as well as sub-samples based on the pre- and post- break periods. More importantly, we find that ignoring structural break when it exists may exaggerate hedging effectiveness.

References

Alsalman, Z., Herrera, A.M., 2013. Oil Price Shocks and the US Stock Market: Do Sign and Size Matter? Paper presented at the 2013 American Economic Association Meetings.

Agren, M., 2006. Does oil price uncertainty transmit to stock market? Working Paper 23, Uppsala University.

Andreou, E., Ghysels, E., 2009. Structural Breaks in Financial Time Series. Handbook of Financial Time Series, 839-870.

Apergis, N., Miller, S.M., 2009. Do structural oil-market shocks affect stock prices? Energy Economics 31, 569–575.

Arouri, M., Jouini, J., Nguyen, D., 2011a.Volatility spillovers between oil prices and stock sector returns: Implications for portfolio management. Journal of International Money and Finance 30, 1387–1405.

Arouri, M., Lahiani, A., Nguyen, D., 2011b. Return and volatility transmission between world oil prices and stock markets of the GCC countries. Economic Modelling 28, 1815 – 1825.

Arouri, M., Nguyen, D., 2010. Oil prices, stock markets and portfolio investment: evidence from sector analysis in Europe over the last decade. Energy Policy 38, 4528 − 4539.

Arouri, M., Jouini, J., Nguyen, D., 2012. On the impacts of oil price fluctuations on European equity markets: Volatility spillover and hedging effectiveness. Energy Economics,34, 611–617.

Balcilar, M., Ozdemir, Z.A., 2012. The causal nexus between oil prices and equity market in the US: A regime switching model. Energy Economics 39, 271 –282.

Barsky, R.B., Kilian, L., 2002. Do we really know that oil caused the great stagflation? A monetary alternative, in: Bernanke, B.S., Rogoff, K. (Eds.), NBER Macroeconomics Annual 2001. MIT Press, Cambridge, MA, 137– 183.

Barsky, R., Kilian, L., 2004. Oil and the macroeconomy since the 1970s.Journal of Economic Perspective 18, 115 –134.

Basher, S., Haug, A., Sadorsky, P., 2012. Oil prices, exchange rates and emerging stock markets. Energy Economics 34, 227–240.

Basher, S., Sadorsky, P., 2006.Oil price risk and emerging stock markets.Global Finance Journal 17, 224– 251.

Bjornland, C.H., 2009. Oil price shocks and stock market booms in an oil exporting country.Scottish Journal of Political Economy 2(5), 232 − 254.

Boyer, M., Filion, D., 2007.Common and fundamental factors in stock returns of Canadian oil and gas companies. Energy Economics 29, 428–453.

Caporin, M., McAleer, M., 2009. Do we really need both BEKK and DCC? A tale of two covariance models. KIER Discussion Paper No. 738.

Colacito, R., Engle, R.F., Ghysels, E., 2011. A component model for dynamic correlations. Journal of Econometrics, 164, 1, 45–59.

Cong, R., Wei, Y., Jiao, J., Fan, Y., 2008. Relationships between oil price shocks and stock market: An empirical analysis from China. Energy Policy 36, 3544– 3553.

Conrad, C., Loch, K., Rittler, D., 2014. On the macroeconomic determinants of long-term volatilities and correlations in US stock and crude oil markets. Journal of Empirical Finance: http://dx.doi.org/10.1016/j.jempfin.2014.03.009.

Elyasiani, E., Mansur, I., Odusami, B., 2012. Oil price shocks and industry stock returns. Energy Economics 33, 966 –974.

El-Sharif, I., Brown, D., Burton, B., Nixon, B., Russell, A., 2005. Evidence on the nature and extent of the relationship between oil prices and equity values in the UK. Energy Economics 27, 819-830.

Engle, R., 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation.Econometrica 50, 987-1007.

Faff, R. W., Brailsford, T. J., 1999. Oil price risk and the Australian stock market.Journal of Energy Finance and Development 4, 69 −87.

Fan, Q., Jahan-Parvar, M.R., 2012.US industry-level returns and oil prices.International Review of Economics and Finance 22, 112– 128.

Fayyad, A., Daly, K., 2011. The impact of oil price shocks on stock market returns: comparing GCC countries with the UK and USA. Emerging Market Review 12, 61 –78.

Fowowe, B., 2013. Jump dynamics in the relationship between oil prices and the stock market: Evidence from Nigeria. http://dx.doi.org/10.1016/j.energy.2013.04.062.

Filis, G., Degiannakis, S., Floros, C., 2011. Dynamic correlation between stock market and oil prices: The case of oil-importing and oil-exporting countries. International Review of Financial Analysis 20, 152–164.

Ghouri, S.S., 2006. Assessment of the relationship between oil prices and US oil stocks. Energy Policy 34, 3327 – 3333.

Hammoudeh, S., Dibooglu, S., Aleisa, E., 2004. Relationships among US oil prices and oil industry equity indices. International Review of Economics and Finance 13, 427-453.

Hammoudeh, S., Li, H., 2005.Oil sensitivity and systematic risk in oil-sensitive stock indices. Journal of Economics and Business 57, 1-21.

Henriques, I., Sadorsky, P., 2008. Oil prices and the stock prices of alternative energy companies.Energy Economics 30, 998-1010.

Huang, R., Masulis, R., Stoll, H., 1996. Energy shocks and financial markets. Journal of Futures Markets 16, 1-27.

International Energy Agency, World Energy Outlook 2008.

Jimenez-Rodriguez, R., Sanchez, M., 2005. Oil price shocks and real GDP growth: empirical evidence for some OECD countries. Applied Economics 37, 201 − 228.

Jones, C., Kaul, G., 1996. Oil and the stock markets.Journal of Finance 51, 463-91.

Jouini, J., Harrathi, N.2014. Revisiting the shock and volatility transmissions among GCC stock and oil markets: A further investigation. Economic Modelling 38, 486–494.

Kang, W., Ratti, R.A., Yoon, K.H., 2014. The impact of oil price shocks on US bond market returns. Energy Economics 44, 248–258.

Kilian, L., Park, C., 2009. The Impact of Oil Price Shocks on the US Stock Market. International Economic Review 50(4), 1267–1287.

Kroner, K., Ng, V., 1998.Modelling asymmetric movements of asset prices. Review of Financial Studies 11, 844 – 871.

Kroner, K., Sultan, J., 1993.Time-varying distributions and dynamic hedging with foreign currency futures. Journal of Financial and Quantitative Analysis 28, 535 – 551.

Ling, S., McAleer, M., 2003.Asymptotic theory for a vector ARMA – GARCH model.Econometric Theory 19, 280– 310.

Malik, F., Ewing, B., 2009.Volatility transmission between oil prices and equity sector returns. International Review of Financial Analysis 18, 95 – 100.

Malik , F., Hammoudeh, S., 2007. Shock and volatility transmission in the oil, US and Gulf equity markets.International Review of Economics and Finance 16, 357− 368.

McAleer , M., Hoti, S., Chan, F., 2009. Structure and asymptotic theory for multivariate asymmetric conditional volatility. Econometric Reviews 28, 422-440.

Mollick, A.V., Assefa, T.A., 2013. US stock returns and oil prices: The tale from daily data and the 2008 – 2009 financial crisis. Energy Economics 36, 1 –18.

Narayan, P., Narayan, S., 2010.Modelling the impact of oil prices on Vietnam's stock prices. Applied Energy 87, 356– 361.

Park, J., Ratti, R., 2008. Oil price shocks and stock markets in the US and 13 European countries.Energy Economics 30, 2587–2608.

Perron, P., 2006. Dealing with structural breaks, in: Palgrave Handbook of Econometrics, Vol. 1, pp 278-352.

Sadorsky, P., 1999. Oil price shocks and stock market activity.Energy Economics 21, 449 − 469.

Schmitz, J., von Ledebur, O., 2011. Approaches to assess higher dimensional price volatility co-movements, in: Piot-Lepetit, I., M'Barek, R. (Eds.), Methods to analyze agricultural commodity price volatility. Springer Science+Business Media, pp. 133-148.

Wang, Y., Wu, C., Yang, L., 2013. Oil price shocks and stock market activities: Evidence from oil-importing and oil-exporting countries. Journal of Comparative Economics 41, 1220–1239.

Appendix A1:

VARMA-BEKK-AGARCH with CCC and DCC VARMA-AGARCH models compared

	BEKK-VARMA-AGARCH		CCC-VARMA-AGARCH		DCC-VARMA-AGARCH	
Mean Equation	USST & Brent	USST & WTI	USST & Brent	USST & WTI	USST & Brent	USST & WTI
ϕ_{10}	-0.007(0.013)	-0.005(0.014)	0.007(0.012)	0.015(0.017)	0.021(0.014)	-0.0008(0.015)
ψ_{11}	-0.061(0.018)*	-0.052(0.017)*	0.045(0.020)**	-0.042(0.020)*	-0.075(0.021)*	-0.046(0.018)**
ψ_{12}	0.002(0.007)	-0.88e-03(0.007)	0.001(0.008)	-0.002(0.008)	0.017(0.004)*	-0.002(0.007)
γ_{11}	0.71e-03(0.000)*	-0.011(0.000)*	0.038(0.000)*	-0.018(0.000)*	0.013(0.000)*	0.026(0.000)*
γ_{12}	-0.002(0.000)*	0.015(0.000)*	-0.036(0.000)*	0.046(0.000)*	0.012(0.000)*	-0.021(0.000)*
φ_{11}	-1.042(0.841)	0.135(0.464)	-1.262(0.026)*	0.467(0.359)	0.035(0.021)	0.487(0.340)
φ_{12}	1.101(0.843)	-0.071(0.463)	1.281(0.027)*	0.461(0.358)	0.003(0.021)	-0.454(0.339)
ϕ_{20}	0.070(0.013)	0.065(0.043)	0.126(0.033)*	0.140(0.048)*	0.137(0.033)*	0.135(0.045)*
ψ_{21}	0.156(0.024)*	0.104(0.030)*	0.142(0.029)*	0.107(0.031)*	0.243(0.243)*	0.103(0.030)*
ψ_{22}	-0.029(0.000)*	-0.035(0.016)**	-0.018(0.020)	-0.033(0.018)	-0.071(0.017)*	-0.042(0.017)**
γ_{21}	-0.006(0.000)*	0.18e-04(0.000)*	-0.061(0.000)*	0.027(0.000)*	0.019(0.000)*	0.041(0.000)*
γ_{22}	-0.007(0.000)*	-0.007(0.000)*	0.005(0.000)*	0.029(0.000)*	0.401(0.000)*	-0.008(0.000)*
φ_{22}	0.445(0.037)*	-0.326(0.674)	-0.680(0.442)	-1.077(0.705)	-0.042(0.045)	-1.032(0.595)
φ_{21}	-0.489(0.006)*	0.279(0.674)	0.531(0.463)	0.909(0.706)	-0.032(0.044)	0.874(0.600)
Variance Equation						
ω_{11}	0.111(0.011)*	-0.105(0.011)*	0.014(0.002)*	0.014(0.003)*	0.022(0.001)*	0.015(0.002)*
ω_{21}	-0.011(0.025)	-0.072(0.033)**	-	-	-	-
ω_{22}	0.104(0.027)*	0.141(0.030)*	0.017(0.007)**	0.052(0.021)**	1.417(0.022)*	0.067(0.024)*
a_{11}	0.084(0.019)*	0.063(0.024)*	-0.022(0.006)*	-0.022(0.007)*	0.088(0.003)*	-0.021(0.006)*
a_{12}	-0.116(0.018)*	-0.191(0.026)*	-	-	-	-
a_{21}	-0.031(0.007)*	-0.032(0.008)*	-	-	-	-

a_{22}	0.160(0.017)*	0.141(0.021)*	0.023(0.005)*	0.042(0.012)*	0.092(0.005)*	0.044(0.012)*
b_{11}	0.955(0.004)*	0.958(0.004)*	0.005(0.009)*	0.929(0.010)*	0.862(0.002)*	0.929(0.009)*
b_{12}	0.010(0.005)	0.007(0.007)	-	-	-	-
b_{21}	0.003(0.002)	-0.13e-03(0.002)	-	-	-	-
b_{22}	0.975(0.003)*	0.972(0.004)*	0.950(0.006)*	0.926(0.014)*	0.562(0.005)*	0.915(0.016)*
c_{11}	0.356(0.019)*	0.353(0.018)*	0.152(0.014)*	0.154(0.015)*	0.067(0.006)*	0.155(0.015)*
c_{12}	-0.011(0.031)	0.011(0.036)	-	-	-	-
c_{21}	0.69e-05(0.010)	0.002(0.036)	-	-	-	-
c_{22}	0.195(0.022)*	0.217(0.008)*	0.045(0.009)*	0.044(0.014)*	0.086(0.006)*	0.055(0.015)*
ρ_{21}	-	-	0.116(0.017)**	0.190(0.016)**		-
θ_1	-	-	-	-	0.014(0.000)*	0.031(0.007)*
θ_2	-	-	-	-	0.986(0.000)*	0.967(0.007)*
Model Diagnostics						
AIC	6.799	6.834	8.841	6.911	N/A	6.826
SBC	6.854	6.889	8.885	6.955	N/A	6.871
Log-L	-10835.9	-10891.52	-10909.55	-11020.83	N/A	-10883.48
Obs.	3196	3196	3196	3196	3196	3196

Residual Diagnostics for Independent Series

	US Stock	Brent	US Stock	WTI	US Stock	Brent	US Stock	WTI	US Stock	Brent	US Stock	WTI
Ljung-Box (20)	14.335	10.267	14.033	17.264	14.795	16.040	14.796	16.040	16.040	24.456	14.747	16.569
Ljung-Box (40)	30.815	33.910	30.884	50.080	32.775	49.393	32.776	49.393	33.885	59.229*	32.577	51.050
McLeod-Li (20)	32.009**	22.061	30.307	22.396	29.111	20.703	29.112	20.703	35.52*	100.7*	28.914	18.567
McLeod-Li (40)	43.147	32.588	40.940	36.234	38.482	32.856	38.483	32.856	46.179	208.924*	38.549	31.159

Source: Compiled by the Authors

Note: Parameters in mean and variance equations are as defined in the model. The US stock is ranked (1) across the model. Brent is ranked (2) in the case of US stock – Brent nexus while WTI is ranked (2) in the case of US stock – WTI nexus. Also, *, ** and *** represent level of significance at 1%, 5% and 10% respectively. Best model is selected based on minimum values of Akaike Information Criterion (AIC) and Swartz Bayesian Criterion (SBC) which are achieved under the maximum log likelihood. Note that AIC and SBC are not comparable for different samples.

Centre for Econometric and Allied Research (CEAR), University of Ibadan, Nigeria.
www.cear.org.ng