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A multi-factor predictive model for oil-US stock nexus with persistence, endogeneity and conditional heteroscedasticity effects

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Abstract

In this study, we extend the single-predictor model for US stock market developed by Narayan and Gupta (2014) to capture more important predictors of the market. Our analyses are conducted in three distinct ways. First, we test whether oil price will produce better forecast accuracy in the multiple-factor model than in the single-factor model. Secondly, we also test the plausibility of making generalization about the predictive model for oil-US stocks on the basis of large cap stocks. Thirdly, we employ the recently developed Feasible Quasi Generalized Least Squares (FQGLS) estimator by Westerlund and Narayan (2014) in order to capture the inherent persistence, endogeneity and heteroscedasticity effects in the predictors. Our results reveal that oil price renders better forecast performance in the multiple-factor predictive model than in the single-factor variants for both in-sample and out-of-sample forecasts. Also, we find that generalizing the predictability of oil-US stock market with large cap may lead to misleading inferences. In addition, it may be necessary to pre-test the predictors for persistence, endogeneity and conditional heteroscedasticity particularly when modeling with high frequency series. Our results are robust to different forecast measures and forecast horizons.

Keywords: WTI Oil price; US large cap; US Small cap; Inflation; Output, Forecast evaluation

JEL Classification: G11, Q43

1. Introduction

Recently, Narayan and Gupta (2014) [NG, thereafter] on the basis of the bias-adjusted OLS estimator by Lewellen (2004) [LW, hereafter] and Feasible Quasi Generalized Least Squares (FQGLS) estimator by Westerlund and Narayan (2012, 2014) [WN, thereafter], find that oil price predicts US stock market in a single-factor predictive regression model. There are a number of attractions to their chosen estimators. First, they account for endogeneity and persistence effects in the predictive model, both of which are prominent features of most economic and financial series. But, in addition, WN (2012, 2014) also modify the LW (2004) to account for conditional heteroscedasticity, which is more likely to be evident in high frequency series.

Meanwhile, there are obvious reasons to believe that including domestic macroeconomic factors in the predictive model for oil-US nexus will offer better or at best enhance the predictive power of any external predictors such as oil price. Hence, in this paper, we revisit the work of NG (2014) in order to account for the role of domestic macroeconomic environment in the forecast of oil-US stock market. Thus, unlike the NG (2014) paper, we propose a multi-factor predictive model for the oil-US stock nexus. In other words, we allow for other predictors that may enhance the forecast performance of NG (2014) model. Motivated by the Arbitrage Pricing Theory (APT), we extend the NG predictive model to include the role of real economic activities (using industrial

production index as a proxy) and macroeconomic uncertainty (using inflation as a proxy). We do acknowledge the significance of oil price used as a single predictor of US stock returns in the NG model. In fact, the literature is replete with empirical evidence in favour of oil price when modeling and forecasting stock returns (See Kilian and Park, 2009; Liu et al., 2015, for a review). However, we argue that the consideration of the macroeconomic environment of the US economy, a reflection of internal economic imbalances, will enhance the forecast performance of the predictive model for the oil-US stock nexus. Moreover, the benchmark crude oil price often used in the forecast model only captures the response of the stock market to external economic imbalances and therefore the forecast performance may be less optimal if oil price constitutes the only predictor of the US stock market as in NG (2014).

Our analyses are conducted in the following distinct ways. First, we examine the superiority of our proposed multiple-factor model over the NG single-factor predictive model in forecasting US stock market. Secondly, we extend the analyses to both large and small cap stocks and therefore, we are able to assess the robustness of our predictive model for different categories of US stocks. Recent studies dealing with US stocks have begun to test the validity of using the large cap stocks to generalize the behavior of different categories of stocks (see for example, Switzer, 2010; Dias, 2013). In addition, we argue that ignoring the fact that the reaction of large cap and small cap to economic and financial risks may vary makes researchers to state inadvertently, that

investors in large cap stocks are faced with similar incidence as investors in small cap which may be misleading. Switzer (2010) examined the relative performance of U.S. and Canada small cap and large cap stocks in the face of economic shocks precisely, under recession and recovery. The study shows that small-cap firms outperform large caps in the year subsequent to an economic trough but tend to lag in the year prior to the business cycle peak. This indicates that the response of small cap and large cap to external shocks may be different.

Thirdly, unlike studies on macroeconomic determinants of US stock market, we also account for endogeneity, persistence and conditional heteroscedasticity in the predictive regression model for the market. As presented in the preliminary analyses in this paper, we find that all the considered predictors exhibit these three statistical properties and therefore the application of the WN (2012, 2014) estimator becomes inevitable. While the literature concerned with the predictability of stock returns is huge, surprisingly little is known when it comes to role of the choice of estimator of the predictive regression (WN, 2012). These authors examine the three mentioned statistical properties which are prominent features of high frequency economic and financial series in order to ascertain whether accounting for them in the estimation process will have any bearing on the forecast performance and their results suggest that it does. To the best of our knowledge, our paper seems to be the first to account for these inherent features of the predictors of US stock market in a multi-factor model set-up.

Consequently, the first objective of this study is to examine whether the US stock market is better predicted with multiple-factor model than the single factor model proposed by NG (2014). This led to the introduction of key macroeconomic variables, that is, US CPI inflation rate and Industrial Production Index. The second objective of this study is to examine whether classification of US stocks into large cap and small cap matters in the predictability of US stocks. In other words, to investigate whether predictions of US large cap stock (S&P 500) could be generalized for small cap stocks (S&P 600). This led to the introduction of S&P 600, which follows an identical model prediction procedure with the S&P 500. In addition, we consider multiple in-sample periods [basically, 50% and 75% of the total observations] and we employ the rolling window approach to generate both the in-sample and out-of-sample forecast results. Also, we employ different forecast measures to evaluate and compare the forecast performance of the competing predictive models.

The remaining part of this paper is structured as follows: Section 2 provides literature review and theoretical background for further justification of the newly introduced variables. Section 3 provides the model set up including the forecast performance measures. Section 4 deals with data and preliminary analysis while empirical result is presented in Section 5. In Section 6, we render some policy implications of the findings and Section 7 concludes the paper.

2. A review of the literature on the predictability of stock market

The research on predictability of stock market from financial and economic covariates has garnered a legion of literature of the past decades (Pierdzioch, 2008; Hjalmarsson, 2010; Nyberg, 2011; Pettenuzzo, et al., 2014; Sun, et al., 2016; Gupta and Wohar, 2017; Nonejad, 2017). The traditional determinants of stock returns include dividend/price ratio, dividend payout ratio, earning price ratio, short term interest rate variable (such as 3-month Treasury Bill Rate) and long term yield are often used in the modeling framework (see Liu et al., 2015; Wu and Lee, 2015). However, existing equilibrium theories between stock returns and fundamentals do not explicitly suggest what and how variables should enter the predictive regressions Liu et al.(2015), and Campbell and Yogo (2006) question the validity of test statistics and strong influences from studies “where autoregressive root of the predictor variable is modelled as a fixed constant less than one” (p.2) . For instance, Liu et al. (2015) examine the predictability of the US stock market with traditional determinants and some macroeconomic variables such as global oil production, global oil demand, crude oil inventory and petroleum consumption. Wu and Lee (2015) conduct similar analysis in respect of simultaneous bear stocks of 10 developed countries including US. They have the objective of examining the relevance of macroeconomic variables in predicting stock returns; hence, they combined the traditional determinants and macroeconomic variables in the model. Their result shows that inflation rate has the highest predictive power among other macroeconomic variables. Similarly, Bu and Pi (2014) proposed to

examine the relevance of investors' sentiment in predicting Chinese stock return. They combine the traditional determinants with investor sentiment index, and find that investors' sentiment has a good predictive power for Chinese stock return. More recently also, Chiang and Hughen (2017) investigate the relevance of future oil price but also combined in the model with the traditional determinants. Though these studies may arrive at a desired result, too many predictors in the model will lead to inclusion of irrelevant variables which will likely lead to in-sample over-fitting problem (Liu et al., 2015).

Meanwhile, NG (2014) does not consider the use of traditional stock market determinants in their oil price single-predictor regression model for predicting US stock market. Though not clearly stated in their work, this approach allows the authors to avoid in-sample over-fitting problem and to conduct a simple but extensive analysis. However, the NG (2014) model may be considered too simple as it only considers oil price (external factor) and ignores the influence of domestic macroeconomic factors (domestic predictors). Our model appreciates the simplicity of the NG (2014) model but avoids being too simplistic by carefully selecting relevant domestic macroeconomic predictors, like real economic activities (using industrial production index as a proxy) and macroeconomic uncertainty (using inflation as a proxy).

2.1 Motivation for the consideration of large- and small-cap stocks

Evidently, large body of literature on the relationship between oil price and stock returns only consider large cap stocks, while ignoring variations in stock market capitalization and its implications. In the case of oil-US stock nexus for instance, the analysis has been between oil price and US S&P 500 (see Kilian and Park, 2009; Mollick and Assefa, 2013; Salisu and Oloko, 2015; Alsalman, 2016; among others). While the mentioned studies focused on in-sample analysis, studies on oil price predictability of stock returns, including NG (2014), also overlook the role of variation in stock market capitalization. However, asset pricing model indicates that large-cap stocks are priced globally while global pricing is rejected for most small-cap stocks (Huang, 2007). This suggests that large-cap stocks is exposed to high level of information and may tend to be more efficient and behave differently compared to small cap stocks.

Meanwhile, some empirical studies have suggested that accounting for variation in stock market capitalization is relevant. On this note, Banz (1981) investigates the relationship between the returns and the total market value of NYSE common stocks. He finds that smaller firms have had higher risk adjusted returns, on average, than larger firms. Similar evidence is reported by Switzer (2010).

Also, Huang (2007) examines cross-country returns correlations and conduct asset pricing tests on three size-based stock portfolios (Large cap and Small Cap) for

nine developed countries. He finds that large cap stock realizes significant co-movements across countries, whereas small-cap stocks realize small average correlations (relative to large cap and mid cap). Again, Diaz (2013) studies the role of market capitalization in the estimation of Value-at-Risk (VaR). He tests the performance of different VaR methodologies for portfolios with different market capitalization. He finds that VaR methods perform differently for portfolio with different market capitalization. In other words, different level of market capitalization reacts differently to a particular VaR method. In predicting stock market returns, oil price may be expected to exhibit better predictive power for large cap stock than for mid and small cap stocks. This is because large cap stock and oil price are internationally priced and therefore tend to be more correlated with oil price than small cap. This is another testable hypothesis in this study.

2.3 Theoretical basis for the predictive model of stock market

The first contribution of this study is to extend the NG (2014) single-factor predictive model to a multiple case. Based on the fact that stock and other financial assets are susceptible to economic and financial risks, this objective is addressed from the viewpoint of the Arbitrage Pricing Theory (APT). Evidently, the APT and the Capital Asset Pricing Model (CAPM) and its variants particularly Fama and French (1993, 1995, 1996, 2015, 2017) are the most prominent asset pricing models in the finance literature. However, our choice of APT is justified based on its ability to estimate the

effect of various macroeconomic and financial risks on stock market; as against the CAPM which only recognizes market risk and Fama and French which deals with firm specific characteristics. In fact, the latter model will require industry level or firm level data (see for example, Narayan and Bannigidadmath, 2015) [NB] and that explains why its application is less popular when forecasting stock prices and returns.

Basically, this study introduces inflation and the real economic activities level into the NG (2014) model. As the basic investment theory proposed, people tend to increase investment when they expect higher inflation rate. This implies that as people expect higher inflation, demand for investment increases, leading to a rise in prices of stocks. Given this transmission, it implies that inflation expectations could help in predicting the current level of stock prices. More so, it has been justified empirically that inflation rate tends to have the highest predictive power (among other macroeconomic variables) in the forecast model for stock market (see Liu et al., 2015).

The second variable introduced into the NG (2014) model is the industrial production index. This is usually used as a measure of real economic activity in a country. The theoretical relationship between stock market and the real economic activity can be explained by the demand-side shock which assumes that a positive productivity shock (as a result of positive demand shock), for instance, leads to higher revenues and profits which causes an increase in dividends and, therefore, stock prices

(see Tsagkanos and Siriopoulos, 2015). Hence, real level economic activity may be expected to predict stock market.

3. The Model and Estimation Procedure

3.1 *Augmented linear and nonlinear covariate models*

Following the literature, the conventional bivariate single predictive regression model in equation (1) postulates oil price as the main predictor of US stock market.

$$r_t = \alpha + \lambda p_{t-1} + \varepsilon_t \quad (1)$$

where r_t and p_t are log of stock prices and oil price, using West Texas Intermediate (WTI) crude oil price, respectively, and ε_t is zero mean idiosyncratic error term on stock returns. The coefficient λ is essentially indicate factor loading on oil prices (Chen et al, 1986; Shanken and Weinstein, 2006). The underlying null hypothesis of no predictability is that $\lambda = 0$. This specification is in line with the NG (2014) for predicting US stock market. Note that equation (1) is a linear version of the single-factor predictive model

However, a number of the extant literature on oil-stock nexus, namely; Kilian and Park (2007), Lee and Chiou (2011), Chang and Yu (2013), Huang et al. (2016), among others, have pointed out that a linear specification of the nexus may not be reasonable since the response of stock market to changes in oil price can be either positive or negative depending on whether the changes are driven by demand- or supply-side shocks. Hence, accommodating this type of nonlinearity in a predictive regression framework becomes imperative. We follow the approach of Shin et al. (2014) to disaggregate the oil price into positive and negative price components as defined below:

$$p_t^+ = \sum_{j=1}^t \Delta p_j^+ = \sum_{j=1}^t \max(\Delta p_j, 0) \quad (2)$$

$$p_t^- = \sum_{j=1}^t \Delta p_j^- = \sum_{j=1}^t \min(\Delta p_j, 0) \quad (3)$$

where p^+ and p^- are defined as positive and negative partial sum decompositions of oil price changes respectively. Given equations (2) and (3), we now have two additional predictors of stock returns, namely, a positive oil price change (p_t^+) and a negative oil price change (p_t^-). Thus, we also examine whether oil price predicts US stock market more accurately when the single-equation predictive regression model is nonlinear as against the linear approach.

$$r_t = \alpha + \lambda^+ p_{t-1}^+ + \varepsilon_t; \quad (4)$$

$$r_t = \alpha + \lambda^- p_t^- + \varepsilon_t; \quad (5)$$

Equations (4) and (5) are the nonlinear variants of the linear single-equation predictive regression model in equation (1). The null hypotheses of no predictability for p_t^+ and p_t^- respectively imply that $\lambda^+ = 0$ and $\lambda^- = 0$.

The proliferation of evidence in favour of the role of macroeconomic environment is enough justification for the inclusion of the two additional predictors in the forecast model for US stock market. Thus, we replace oil price with industrial production index and inflation singly in equation (1). We utilize monthly US industrial production index as a proxy for output in the alternative single-factor predictive model for stock market.

$$r_t = \alpha + \beta y_{t-1} + \varepsilon_t \quad (6)$$

where y_t denotes US industrial production index, while $\beta = 0$ is the null hypothesis of no predictability in the case of real economic activities. In a similar fashion, we use monthly CPI Inflation as the second alternative predictor of stock price. The proposed predictive single-factor model in this regard is as below;

$$r_t = \alpha + \delta \pi_{t-1} + \varepsilon_t; \quad H_0 : \delta = 0 \quad (7)$$

where π_t is the log of headline CPI.

We further augment oil price with each of the proposed alternative predictors in the respective single-factor predictive models and then, we arrive at the following multiple-factor predictive models for US stock market as below.

Symmetric version:
$$r_t = \alpha + \lambda p_{t-1} + \beta y_{t-1} + \delta \pi_{t-1} + \varepsilon_t \quad (8a)$$

Asymmetric version:

$$r_t = \alpha + \lambda p_{t-1}^+ + \beta y_{t-1} + \delta \pi_{t-1} + \varepsilon_t \quad (8b)$$

$$r_t = \alpha + \lambda p_{t-1}^- + \beta y_{t-1} + \delta \pi_{t-1} + \varepsilon_t \quad (8c)$$

The motive for the above multiple-factor predictive models is to find out whether augmenting the oil price with the alternative predictors proposed will produce a better forecast performance than any of the single-factor variants. Note that equation (8a) involves the symmetric oil price while equations (8b) and (8c) capture the asymmetric-type models for positive and negative oil price changes respectively. If the proposed multiple-factor predictive model for the US stock market is valid, then, null hypotheses for no predictability for real economic activities and inflation should be rejected under the three scenarios as in equations (8a), (8b) and (8c).

In addition, as previously noted, recent development in single-equation forecasting models seems to favour the consideration of persistence and endogeneity

issues (see LW, 2004; NG, 2014; NB, 2015).¹ These statistical features have implications on the choice of estimator for forecasting which consequently affect the outcome of the forecast results.² LW (2004) demonstrates the procedure for the adjustment of the OLS slope coefficient of the predictor in order to correct for the bias introduced by the presence of persistence and endogeneity in the model. Similar applications are demonstrated in the works of NG (2014) and NB (2015). Thus, we further test whether our predictors truly exhibit these features and if they did, the predictive regression model will have to be estimated with the bias-adjusted OLS estimator proposed by LW (2004). More so, WN (2012, 2014) propose an extension of the LW (2004) which accounts for conditional heteroscedasticity. This may be more appropriate since we are dealing with financial series such as stock prices that are available at high frequency and are therefore more susceptible to volatility (see NB, 2015).

3.2 Measures of forecast performance

For robustness purpose, we consider multiple in-sample periods using 50% and 75% of total observations. The use of multiple in-sample periods has become a standard practice for evaluating the forecast performance of predictive regression models (see for example, Welch and Goyal, 2008; Rapach et al., 2010, among others). Also, we employ

¹Also, financial series that are usually available at a high frequency (such as intra-day, daily and weekly) tend to exhibit conditional heteroscedasticity which must be accounted for in the forecasting model (see Westerlund and Narayan, 2012, 2014).

² In the appendix, we offer theoretical expositions for the implementation of the WN estimator for both single-factor and multi-factor model set-up.

the rolling window approach to generate the forecasts and this approach is preferred to the fixed parameter method as the former accounts for the time varying nature of the estimates. This is particularly important when dealing with long time series that may have witnessed structural changes over time. The rolling window approach estimates one step ahead, then drops an observation at the start and adds the new observation to the sample and thereafter re-estimates. This process is followed continuously until the end of the forecast period is reached.

Also, we employ the Root Mean Square Error (RMSE) in order to measure the predictive ability of the various predictors. These measures are computed for both the in-sample forecast and out-of-sample forecast. We further consider the Campbell and Thompson (2008) alternative measure of forecast performance described as the out-of-sample R^2 (OOS_R) statistic. The OOS_R is given by $OOS_R = 1 - (M\hat{S}E_1 / M\hat{S}E_0)$, where $M\hat{S}E_1$ and $M\hat{S}E_0$ are the mean square error (MSE) of the out-of-sample prediction from the restricted and unrestricted models, respectively. Hence, the restricted model in the context of this study is the single-factor variants while the unrestricted version is the multiple-factor models. A positive value of the statistic i.e. $OOS_R > 0$ suggests that the unrestricted model outperforms the restricted model; otherwise, it does not.

4.0 Data and Preliminary Analyses

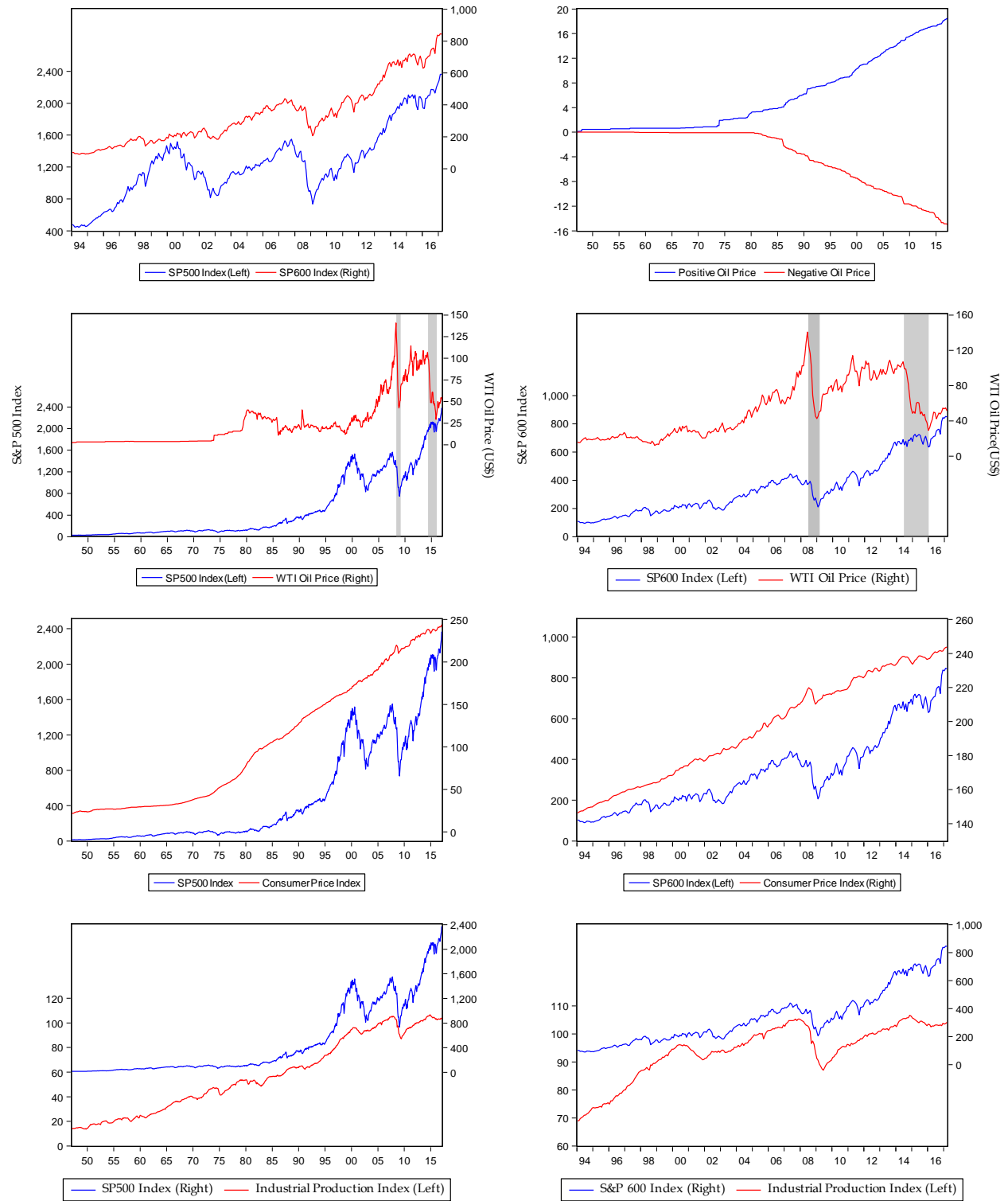
4.1 Data Issues

Basically, to make a baseline case of the single-factor predictive model by NG (2014), US S&P 500 index and WTI oil price are employed, and Shin et al. (2014) method is used to decompose oil price into positive and negative oil price. Hence, this study makes use of monthly data for WTI oil price, US S&P 500 index (large cap stocks), US S&P 600 index (small cap stocks), and US CPI Inflation rate and US Industrial Production Index (IPI). These are obtained from Bloomberg Terminal. The periods of analysis for large cap and small cap stocks however differ due to data availability constraint. Specifically, the analysis of large cap stocks covers from Jan, 1947 to March, 2017, while that of small cap stocks covers from Jan 1994 to March, 2017. Thus, we have 843 and 279 observations for large cap and small cap stocks, respectively; each of which is large enough to make reasonable inference.

Figure 1 below shows the graphical presentation of the data employed for the analysis. In the first row of the figure are the graphs for large and small cap stocks, and positive and negative WTI oil price. In the second row is the bi-directional graph of the large cap - oil price and small cap - oil price relationship. The two periods highlighted on the graphs relate to the period of global financial crisis (GFC) and the oil price crash of the 2014. It is evident that stock prices crashed as oil price crashed during the GFC and also sluggish when oil price crashed in 2014. This indicates some level of

relationship pointing to the fact that oil price could potentially drive or predict US stock price. In the third row of the figure, domestic macroeconomic variables (US CPI inflation and the real level of economic activities) are presented. Their trends with the stock price are similar to that of oil price. Thus, extending the single-factor predictive model with these variables is expected to enhance its forecast accuracy.

Figure 1: Graphical presentation of variables



4.2 Descriptive Statistics

Table 1 presents the results of the selected descriptive statistics for variables in both large cap and small cap stock models. The descriptive statistics of variables in the large cap stock model are presented in Panel 1A of the table, while same are presented for small cap stock model in Panel 1B. From both Panels 1A and 1B, it is observed from the standard deviation coefficient that stock prices are the most volatile series while the results of skewness and kurtosis panels are mixed for the two categories. Nonetheless, all the series are non-normal as expected for most high frequency financial and economic series. Thus, in the determination of the weight used in transforming the original predictive model, we consider a non-normal error distribution which mimics the distribution of the regression error in order to produce more precise and accurate forecasts.

Table 1: Selected Descriptive Statistics

Variables	Initials	Mean	Std. Dev.	Skewness	Kurtosis	J-B
<i>Panel 1A: Large cap stocks model</i>						
Large cap stock	s_t^{lc}	494.287	592.908	1.215	3.304	210.808***
Oil Price	p_t	24.311	27.522	1.623	4.986	508.669***
Inflation	π_t	104.574	73.935	0.428	1.717	83.534***
Output level	y_t	57.329	29.714	0.212	1.702	65.523***
<i>Panel 1B: Small cap stocks model</i>						
Small cap stock	s_t^{sc}	337.038	189.087	0.818	2.743	31.736***
Oil Price	p_t	51.456	30.981	0.546	2.053	24.191***

CPI	π_t	196.471	29.942	-0.025	1.618	22.156***
IPI	y_t	94.153	9.443	-0.971	3.133	43.876***

Note: *** indicates 1% level of significance.

4.3 Preliminary Analyses

We carry out preliminary tests for the presence of significant autocorrelation, heteroscedasticity, persistence and endogeneity effects in the predictors in large cap and small cap stock models. All variables are now expressed in natural logarithm and oil price is decomposed into negative and positive oil price to account for non-linearity in oil price effect. The results for Ljung-Box residual autocorrelation test and the test for ARCH effect are presented in Table 2, while the results for unit root, persistence and endogeneity tests are presented in Table 3.

Table 2: Autocorrelation & ARCH tests for the predictors

Predictors	Autocorrelation (Q-stat)			Heteroscedasticity test (ARCH-LM test)		
	$k = 1$	$k = 6$	$k = 12$	$k = 1$	$k = 6$	$k = 12$
<i>Panel 2A: Large cap stocks model</i>						
p_t	18.236***	23.571***	34.086***	6.559**	2.445**	1.285
p_t^+	6.764***	16.261**	26.811***	0.144	0.211	0.116
p_t^-	98.410***	126.11***	135.78***	227.991***	39.379***	19.643***
π_t	198.87***	486.57***	878.85***	137.713***	36.318***	23.166***
y_t	122.00***	232.63***	267.28***	105.900***	20.949***	10.554***
<i>Panel 2B: Small cap stocks model</i>						
p_t	273.70***	1515.7***	2804.9***	2227.616***	377.4790***	194.6841***
p_t^+	275.51***	1586.4***	3017.0***	130055.7***	28083.13***	14348.03***
p_t^-	274.81***	1569.6***	2949.5***	101811.7***	19073.47***	10768.96***
π_t	276.32***	1588.0***	3016.9***	109641.4***	25369.22***	12655.05***

y_t	271.99***	1488.3***	2620.8***	78663.86***	10442.81***	3898.492***
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Note: The reported values are the F-statistics of the respective test. k indicates the chosen lag length for the tests. *** and ** indicate 1% and 5% level of significance, respectively.

From Table 2, it is revealed from Panel 2A that all the predictors in the large cap stock model exhibit high level of autocorrelation, as the null hypothesis of no autocorrelation is rejected at 1 percent across different lag periods. Also, there is evidence of strong autoregressive conditional heteroscedasticity (ARCH) for negative oil price, inflation and output level, weak ARCH effect for oil price and no ARCH effect for positive oil price. In Panel 2B however, it is evident that the null hypothesis of no Autocorrelation and no ARCH is strongly rejected for all the predictors in the small cap stock model. For both large and small cap stock models, the result suggests that virtually all the predictors exhibit autocorrelation and ARCH effects; hence, the choice of an estimator consistent with autocorrelation and conditional heteroscedasticity is important.

Table 3: Preliminary Tests

Variables	ADF Unit root		Persistence	Endogeneity
	T-stat [k]	I (d)		
<i>Panel 3A: Large cap stocks model</i>				
s_t^{lc}	-27.900[0]***	I (1)		
p_t	-24.983[0]***	I (1)	0.9974***	0.4059
p_t^+	-25.052[0]***	I (1)	1.0022***	1.0239***
p_t^-	-18.847[0]***	I (1)	1.0030***	-1.8539***
π_t	-4.366[10]***	I (1)	0.9997***	-21.6388***
y_t	-11.481[2]***	I (1)	0.9983***	4.0519***

s_t^{sc}	-15.554[0]***	I (1)		
p_t	-14.787[0]***	I (1)	0.9855***	0.2365
p_t^+	-16.247[0]***	I (1)	0.9996***	0.0392
p_t^-	-13.288[0]***	I (1)	1.0016***	0.0881
π_t	-10.858[1]***	I (1)	0.9977***	6.9743**
y_t	-4.179[3]***	I (1)	0.9856***	5.6704**

Note: k represents the optimal lag for the ADF test selected using Akaike Information Criterion (AIC), and *** indicates 1% level of significance.

Furthermore, Table 3 presents the results for unit root, persistence and endogeneity tests. In Panel 3A which presents the corresponding results for variables in the large cap model, the Augmented Dickey-Fuller (ADF) unit root result shows that all the variables are integrated of order 1. This is also true for all the variables in the small cap stock model in Panel 3B. However, as stationarity does not imply persistence, we further conduct persistence test for the predictors in the two models. The result for persistence shows that the predictors have high persistence, as they are very close or equal to unity. Meanwhile, the result for endogeneity test shows that inflation and output level are endogenous in both large cap and small cap models, and in addition, positive and negative oil price are endogenous in the large cap model. This suggests that an estimator consistent with endogeneity must be chosen to avoid endogeneity bias in the empirical results. The competing estimators are LW (2004) and WN (2012), however, to further account for heteroscedasticity in the series as noted previously, WN (2012) is favoured.

5.0 Discussion of Results

As previously mentioned, we favour the WN estimator on the basis of the underlying statistical properties of the predictors considered in the predictive model for the US stock prices. For clarity and robustness, we partition our analyses as follows. We first establish the predictability of the potential predictors of stock prices for both large and small cap stocks by reporting the bias-adjusted GLS estimate for each factor in each model (single-factor and multiple-factor models). The analyses are performed for both 50% and 75% of the observations. Secondly, we evaluate the forecast accuracy of the in-sample estimates using the root mean square error as well as the use of plots to show graphically how each of the models is able to track actual stock price indexes. Thereafter, we further subject the models to out-of-sample forecast analyses since the in-sample forecast performance does not entirely guarantee accurate out-of-sample forecast results. Put differently, the best model for in-sample forecasts may be less optimal when subjected to out-of-sample evaluation. Following the highlighted procedure, we begin our discussion of results with in-sample forecast and afterwards, we extend to out-of-sample forecast.

5.1 *In-Sample forecast performance: the predictability results for large and small cap stocks*

The predictability results for both large and small cap stocks are presented in Table 4. The results highlight the significance of each of the predictors in the various forecast models. As shown in the table, virtually all the predictors are statistically significant at the 1 per cent level, thus, rejecting the null hypothesis of no predictability for oil price, output and inflation. This evidence is not only valid for large cap but also for small cap. The finding also suggests that whether using small or large sample, the conclusion remains the same. This is quite instructive for a number of reasons. It is an indication that US stock market responds significantly to both external and internal economic imbalances (see also Angelidis et al., 2015). The external imbalances are usually captured with oil price since it responds to both global demand and supply shocks (see Kilian, 2009). Also, internal macroeconomic imbalances are usually measured using output variability and inflation. All these variables which are reflected both individually and jointly are good predictors of the US stock market. No doubt, the literature is replete with evidence of significant role of oil price in predicting US stock market (see for example, Kilian and Park, 2009; Narayan and Gupta, 2014; Salisu and Oloko, 2015). What is scarce in the literature however, is the possibility of joint significance of all the variables considered in the prediction of US large and small cap stocks while also accounting for persistence, endogeneity and conditional heteroscedasticity. We may be tempted to argue that the inability of the previous studies to account for these inherent statistical properties of the predictors may be responsible for lack of empirical support for joint significance. This temptation may not

be out of place judging by the results of Campbell and Yogo (2006), Hjalmarsson (2010) and WN (2012) which suggest that the choice of estimator matters when forecasting stock market. These studies argue that the choice of estimator should be rooted in the salient features of the data.

Let us now turn to the single equation case. On average, the level of real economic activities seems to exert the biggest impact on stock returns followed by inflation judging by the magnitude of the regression coefficients. Thus, the US stock market tends to respond more to internal economic imbalances than external. We also find that both cap stocks respond asymmetrically to changes in oil price since the estimates for both negative and positive oil price changes are statistically significant.

Table 4: Predictability results for oil-US stock nexus

Predictor	Large Cap		Small Cap		
	50%	75%	50%	75%	
<i>Single factor</i>					
P_t	0.460959*** (0.032611)	0.810993*** (0.028996)	0.727091*** (0.056485)	0.608813*** (0.023698)	
P_t^+	0.456134*** (0.030563)	0.342847*** (0.006348)	0.193041*** (0.007232)	0.153006*** (0.004982)	
P_t^-	-19.33031*** (0.474484)	-0.656404*** (0.040567)	-0.363751*** (0.034241)	-0.387488*** (0.020517)	
π_t	1.401177*** (0.063650)	1.475136*** (0.024431)	4.121665*** (0.121631)	3.219116*** (0.091943)	
y_t	1.444865*** (0.026875)	1.970289*** (0.026248)	3.059516*** (0.102371)	3.681725*** (0.123971)	
<i>Multiple factor</i>					
Case I	P_t	0.034616 (0.065216)	-0.574994*** (0.022121)	0.043530 (0.048464)	0.133408*** (0.034067)
	y_t	2.025551***	1.380907***	1.404548***	1.901418***

		(0.083947)	(0.044331)	(0.226645)	(0.110648)
	π_t	-0.687138*** (0.207805)	1.258594*** (0.047310)	2.281545*** (0.392755)	1.380890*** (0.193828)
Case II	p_t^+	0.133021* (0.068007)	0.262947*** (0.008644)	-0.166995*** (0.047032)	-0.040541 (0.035569)
	y_t	2.113066*** (0.083798)	2.004737*** (0.042236)	0.920797*** (0.226100)	1.749361*** (0.129027)
	π_t	-0.996546*** (0.221268)	-0.983848*** (0.053443)	6.449167*** (1.134744)	2.931125*** (0.782580)
Case III	p_t^-	-5.952692*** (0.680406)	-0.249090*** (0.020005)	0.271920*** (0.064092)	0.118007*** (0.033117)
	y_t	1.580602*** (0.069376)	1.866130*** (0.040207)	1.799858*** (0.403204)	2.101220*** (0.261812)
	π_t	-0.660436*** (0.061109)	-0.414235*** (0.041701)	5.236515*** (0.944623)	2.977962*** (0.477552)

Note that values in parentheses () denote standard errors.

Like the single-factor model, virtually all the variables are statistically significant in the multiple-predictor case particularly when we account for asymmetries. Thus, regardless of the choice of predictive model, the factors considered are important predictors of both US large and small cap stocks. The next objective is to evaluate the forecast performance of both the single-factor and multiple-factor models.

5.2 In-sample forecast evaluation

Which of these two model variants will produce better in-sample forecasts for the US stock market? This question becomes necessary since the null hypothesis of no predictability is rejected for all the variables and for both single-factor case and multiple-factor case regardless of the choice of in-sample data scope. To answer this question, we compute the root mean square error (RMSE) for each of the predictive

model variants and we also produce graphs which plot each of the fitted values against the actual data. The RMSE values are reported in Table 5 while the plotted figures are presented afterwards.

Looking at the single-factor case, the in-sample forecast results depict that the predictive model with the factor of real economic activities produces the least RMSE values for large cap stocks while inflation appears to give the lowest RMSE values for small cap stock irrespective of the in-sample period. Thus, if we were to use a single-factor model to forecast the US stock, the output-factor model may be preferred to the oil-based US stock model for large cap stocks. However, the inflation-factor predictive model is more likely to produce better forecast results for the small cap stocks. These results seem consistent with the predictability results of the FQGLS estimator reported in Table 4. What does this result imply? The implication is that it will be erroneous to use the predictive model for large cap stocks to generalize for small cap stocks. It also suggests that oil price-based single factor model underperforms those that capture the role of macroeconomic environment and therefore may not be adequate in predicting US stock market whether large or small.

Coming back to the initial question, when we compare the single-factor model variants with the multiple-factor, we find that the latter outperforms the former regardless of the variants, cap stock and in-sample period. In other words, any of the

variants of the multiple-factor predictive model will produce more accurate in-sample forecasts for the US stock market (whether large or small) than the single-factor variants. This is an indication that the forecast performance of the predictive model for oil-stock nexus can be enhanced when the role of macroeconomic environment is not jettisoned. This finding is further justified graphically. A closer look at the various plots for large cap stock (see Figures 2 and 3) and small cap stock (see Figures 4 and 5) suggests that the multiple-factor model appears to track/fit the actual data more accurately than the single-factor variants.³ This is evident over the entire in-sample periods for the two cap stocks. More specifically, figures 2.4 and 3.4 for 50% and 75% respectively track better than other figures for the large cap stocks. Similar trends are observed for small cap stocks when we compare figures 4.4 and 5.4 with other figures for 50% and 75% respectively.

Drawing from the foregoing, it may be necessary for financial analysts and investors seeking to maximize their investment returns in the US stock market to consider the following in order to produce reasonably accurate in-sample forecasts. First, the predictors of US stocks seem to exhibit both persistence and endogeneity effects which have to be dealt with in the predictive model. Ignoring these effects will bias the regression estimates for the predictors. Secondly, if high frequency series are

³ The plots presented are generated from the symmetric variants of both the single-factor and multiple-factor models. Although, the asymmetric variants are suppressed here for brevity, the conclusion remains the same with or without the asymmetric variants.

being used as predictors, then, it becomes necessary to test and account for conditional heteroscedasticity (if it exists) in the estimation of the predictors. The GLS-based t statistic estimator by WN (2012) is found to be relevant in this regard. However, if there is no evidence of conditional heteroscedasticity, then the LW estimator may be employed. Thirdly, while the multiple-factor model is suitable for both large and small cap stocks, these stocks however respond differently to the single-factor variants. Therefore, where a modeler decides to forecast with a single-factor model (as in the case of NG, 2014), then, some caution should be exercised in terms of the choice of predictor for the two cap stocks. As demonstrated, using the same single-predictor model to forecast both cap stocks may produce less desirable results. Lastly, the multiple-factor predictive model that accounts for both internal and external economic imbalances is crucial for forecasting US stocks.

Table 5: In-sample forecast (RMSE)

Predictor	Large cap		Small cap	
	50%	75%	50%	75%
<i>Single factor</i>				
p_t	0.5337	0.7218	0.2340	0.2136
p_t^+	0.5222	0.4446	0.1379	0.1849
p_t^-	0.2618	0.5387	0.1287	0.1624
π_t	0.4244	0.4138	0.1135	0.1656
y_t	0.2306	0.3409	0.1271	0.1919
<i>Multiple factor</i>				

$p_t - y_t - \pi_t$	0.1890	0.1996	0.0971	0.1013
$p_t^+ - y_t - \pi_t$	0.1884	0.1827	0.0926	0.1040
$p_t^- - y_t - \pi_t$	0.1693	0.1701	0.0897	0.1013

Figure 2: In-sample forecast performance for large cap (50%)

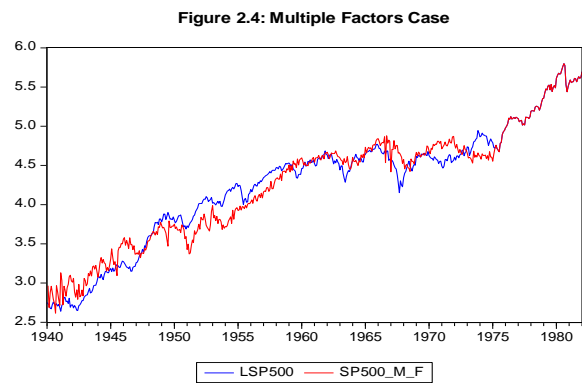
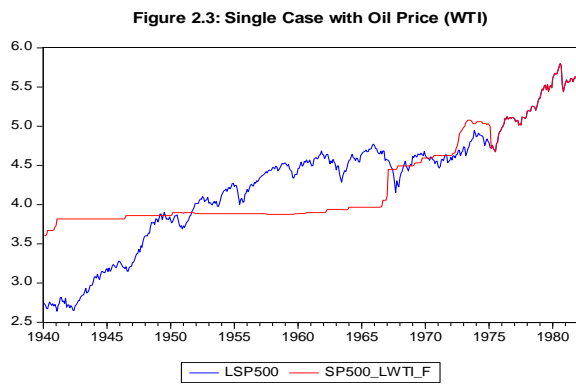
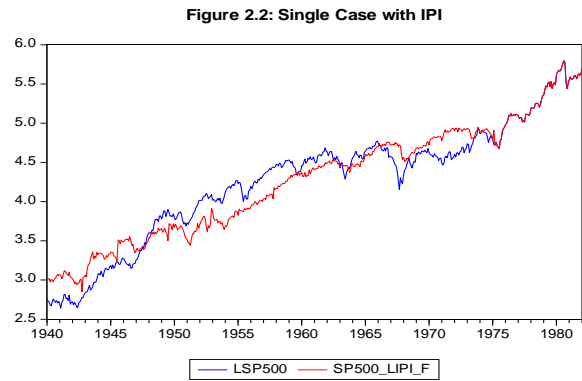
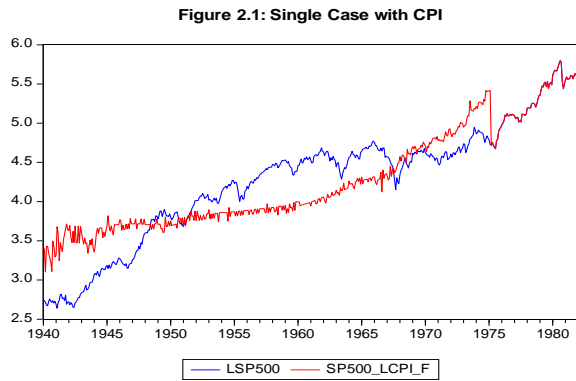


Figure 3: In-sample forecast performance for large cap (75%)

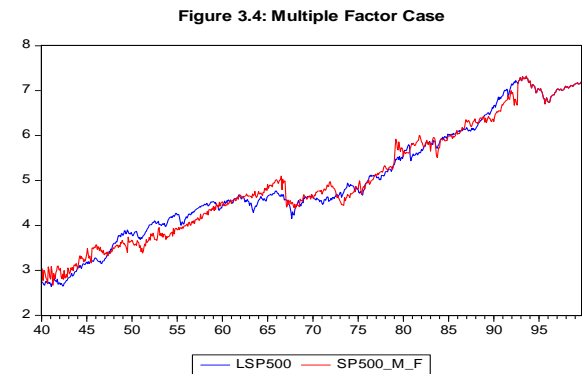
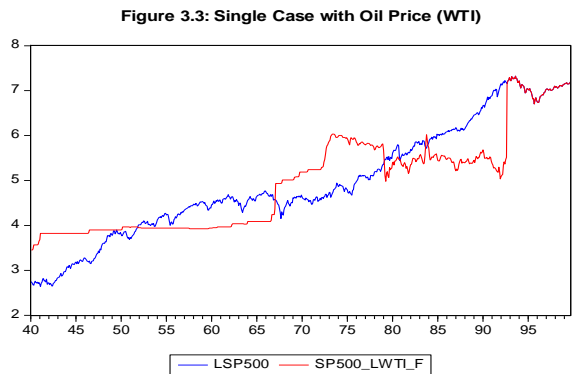
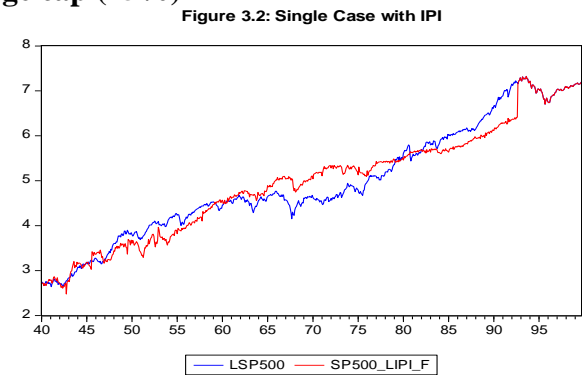
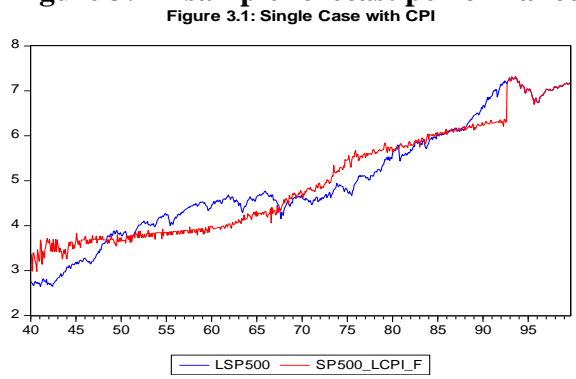


Figure 4: In-sample forecast performance for small cap (50%)

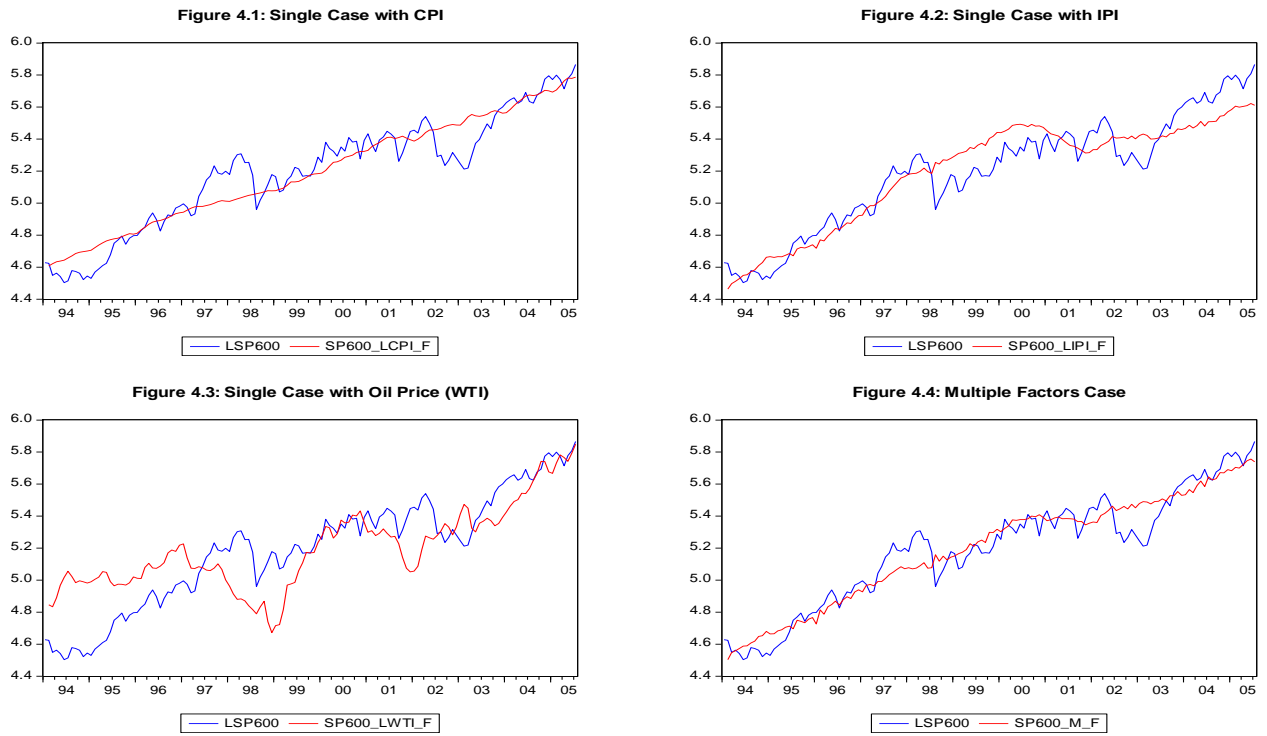
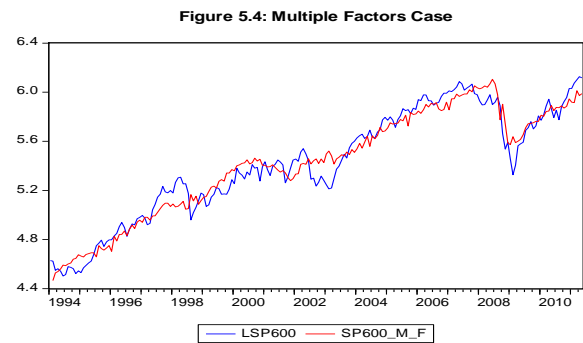
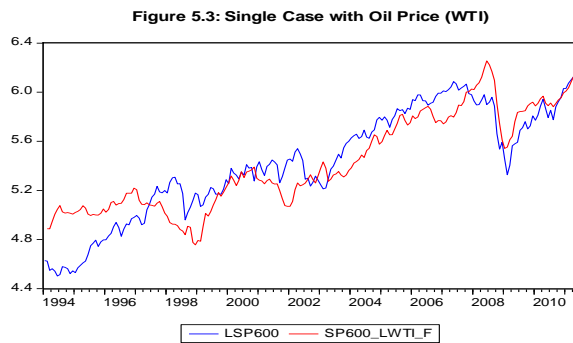
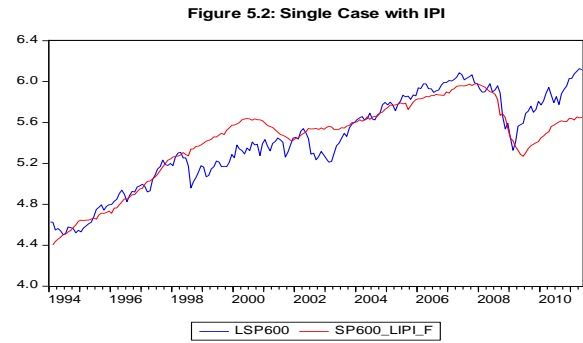
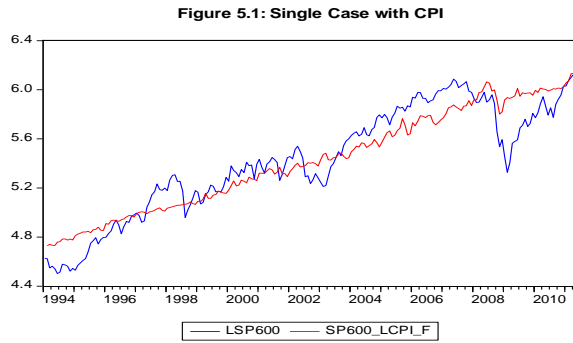


Figure 5: In-sample forecast performance for small cap (75%)



5.3 Out-of-Sample forecast performance

Since better in-sample forecast does not automatically guarantee a better out-of-sample forecast, we now consider the out-of-sample forecast performance of the models. Basically, this allows us to compare the single factor oil price-US stock predictive model by NG (2014) with our proposed multiple-factor predictive model. The basis for evaluating and comparing the forecast performance of two competitive models in this study is the Campbell and Thompson (2008) method. This method makes decision on the superiority or otherwise of the unrestricted model (multiple factor) based on the Mean Square Error (MSE) of both the restricted (single-factor) and the unrestricted (multiple-factor) models.

Forecasting is conducted in this study using rolling window approach. For robustness check however, we forecast using 50% and 75% of the available data and for 6-months ($h=6$), 12- months ($h=12$), and 18-months ($h=18$) ahead forecast horizons. This approach is used to evaluate the forecast performance of the single-case and multiple-case small and large cap stocks models.

5.3.1 Single-predictor out-of-sample forecast: Large Cap and Small Cap Models

We examine oil price predictability of large cap stock under single-factor and multiple-factor predictive models. More so, combined factor model is considered, where the predictive power of relevant single-predictor models is averaged. This approach requires that the RMSE of relevant single-predictor models is averaged compared to the case of multiple-factor model where a unique RMSE is determined by multiple predictors in a regression model. The comparison is done to examine whether combined forecast approach will outperform the multiple-factor model in predicting US stock market.

Table 6 presents the out-of-sample forecast performance of large cap and small cap models. This is represented by the RMSE statistics, which is a measure of variation between the actual data and the generated forecast data. The optimal value for RMSE is zero, and so, the closer the RMSE of a model to zero, the better. Recall that the single-factor model presents stock prices as a function of each of the predictors, hence, the result is displayed for the RMSE when stock price is singly presented as a function of oil

price, positive oil price, negative oil price, inflation and output level, in that order. Similarly, the values presented for the multiple factor model is the RMSE obtained when stock price is expressed as a function of oil price, inflation and output level⁴. More so, the values presented for combined forecast is the average RMSE of the relevant single factor models.

⁴ Note that alternative scenarios are created by disaggregating oil price changes into positive and negative; in recognition of the non-linear oil price effect.

Table 6: Out-of-sample forecast performance

Predictor	Large Cap Stocks						Small Cap stocks					
	50%			75%			50%			75%		
	$h = 6$	$h = 12$	$h = 18$	$h = 6$	$h = 12$	$h = 18$	$h = 6$	$h = 12$	$h = 18$	$h = 6$	$h = 12$	$h = 18$
<i>Single factor model</i>												
P_t	0.5309	0.5274	0.5241	0.7342	0.7448	0.7529	0.2293	0.2254	0.2220	0.2107	0.2081	0.2063
P_t^+	0.5214	0.5192	0.5161	0.4426	0.4405	0.4393	0.1354	0.1334	0.1317	0.1841	0.1819	0.1798
P_t^-	0.5096	1.0907	2.0104	0.5385	0.5385	0.5375	0.1353	0.1436	0.1532	0.1616	0.1639	0.1674
π_t	0.4291	0.4318	0.4312	0.4217	0.4289	0.4336	0.1116	0.1101	0.1085	0.1650	0.1632	0.1613
y_t	0.2293	0.2284	0.2284	0.3479	0.3549	0.3601	0.1348	0.1419	0.1495	0.1955	0.2003	0.2049
<i>Multiple factor model</i>												
$P_t - y_t - \pi_t$	0.1899	0.1945	0.2043	0.2087	0.2184	0.2248	0.0971	0.0967	0.0969	0.1002	0.0992	0.0984
$P_t^+ - y_t - \pi_t$	0.1893	0.1938	0.2028	0.1820	0.1818	0.1835	0.0926	0.0919	0.0916	0.1032	0.1020	0.1009
$P_t^- - y_t - \pi_t$	0.1767	0.2428	0.3918	0.1701	0.1697	0.1691	0.0889	0.0897	0.0891	0.1009	0.0997	0.0985
<i>Combined forecast</i>												
$P_t - y_t - \pi_t$	0.3964	0.3959	0.3945	0.5013	0.5095	0.5155	0.1586	0.1591	0.1600	0.1904	0.1905	0.1909
$P_t^+ - y_t - \pi_t$	0.3933	0.3931	0.3919	0.4041	0.4081	0.4110	0.1273	0.1285	0.1299	0.1815	0.1818	0.1820
$P_t^- - y_t - \pi_t$	0.3893	0.5836	0.8900	0.4361	0.4408	0.4438	0.1272	0.1319	0.1371	0.1740	0.1758	0.1779

Source: Computed by the authors

Note: The values reported are the RMSEs from the single, multiple and combined factor models. The lowest RMSE values under each forecast horizon and in-sample period are highlighted.

From the single factor model for large cap stocks, it is evident that the level of real economic activities is the best predictor for the large cap stocks. Consistently, this variant of the single-factor models produces the least RMSE values regardless of the in-sample periods and forecast horizons. However, for the small cap stocks, inflation seems to be the best predictor. This finding further reinvigorates our hypothesis that the generalization of predictive regression model for US stocks with large cap stocks may not be entirely valid. In terms of asymmetries, we find that large cap stocks are better predicted by positive oil price changes (p_t^+) relative to the negative (p_t^-) and symmetric (p_t) oil prices. Conversely however, the small cap stocks respond more to the negative oil price changes (p_t^-) than the positive (p_t^+) and symmetric (p_t) oil prices. This suggests that when modeling oil-stock relationship, it may be necessary to account for non-linearity distinctly for large and small cap stocks.

5.3.2 Multiple-predictor out-of-sample forecast: Large Cap and Small Cap Model

Multiple-predictor model presents the RMSE produced when stock prices are expressed as a function of multiple factors; that is, oil price and domestic macroeconomic variables, inflation and the level real economic activities. From the Table 6, it is evident that the forecast performance of both large cap and small cap stocks under the multiple-factor case improved significantly relative to the forecast performance under the single-factor models. Lower RMSE values are consistently

recorded for the multiple-factor variants over the different forecast horizons and different in-sample periods. More importantly, irrespective of the choice of multiple-predictor model, superior out-of-sample forecast performance is achieved even relative to the best single-predictor variant. This finding suggests that US stocks whether large or small are better predicted with a multiple-factor model. Nonetheless, in terms of the comparative performance of the multiple-factor variants, while the inclusion of both inflation and output matters in the predictive model for large and small stocks, the choice of symmetric or asymmetric oil price matters for the out-of-sample forecast results

The superiority of multiple-factor models over the single-factor model is further explained by the positive values of Campbell-Thompson (C-T) test under Cases I to III in the Table 7. Recall that the C-T test compares the unrestricted (multiple-factor) model with the restricted (single-factor) model, hence positive value indicates better performance of the unrestricted model (see NG, 2014). The results in Cases I to III clearly support the use of a multiple-factor model that accounts for internal economic imbalances in the prediction of the US stock market. This result still holds even when oil price is disaggregated into positive and negative in the single-factor model. Furthermore, as noticed from in-sample and out-of-sample forecast for single predictor models, the result reveals that it is important to consider non-linearity in oil price when predicting large cap and small stocks in a single-factor model.

Table 7: Results of Campbell- Thompson test

Predictor	Large Cap Stocks						Small Cap Stocks					
	50%			75%			50%			75%		
	$h = 6$	$h = 12$	$h = 18$	$h = 6$	$h = 12$	$h = 18$	$h = 6$	$h = 12$	$h = 18$	$h = 6$	$h = 12$	$h = 18$
Case I												
$m^{p_t} - s^{p_t}$	0.6422	0.6313	0.6101	0.7157	0.7068	0.7014	0.5765	0.5710	0.5638	0.5245	0.5235	0.5233
$m^{p_t} - s^{p_t^-}$	0.6272	0.8217	0.8984	0.6125	0.5945	0.5818	0.2822	0.3268	0.3677	0.3799	0.3951	0.4124
$m^{p_t} - s^{p_t^+}$	0.6357	0.6254	0.6041	0.5284	0.5043	0.4883	0.2830	0.2751	0.2646	0.4557	0.4550	0.4530
Case II												
$m^{p_t^+} - s^{p_t}$	0.6435	0.6325	0.6130	0.7521	0.7559	0.7563	0.5963	0.5923	0.5876	0.5102	0.5098	0.5111
$m^{p_t^+} - s^{p_t^-}$	0.6286	0.8223	0.8991	0.6620	0.6625	0.6587	0.3157	0.3603	0.4022	0.3612	0.3778	0.3973
$m^{p_t^+} - s^{p_t^+}$	0.6370	0.6267	0.6070	0.5887	0.5874	0.5824	0.3164	0.3112	0.3048	0.4393	0.4394	0.4390
Case III												
$m^{p_t^-} - s^{p_t}$	0.6672	0.5397	0.2524	0.7683	0.7722	0.7754	0.6125	0.6020	0.5986	0.5212	0.5206	0.5227
$m^{p_t^-} - s^{p_t^-}$	0.6533	0.7774	0.8051	0.6842	0.6849	0.6855	0.3431	0.3755	0.4181	0.3756	0.3915	0.4117
$m^{p_t^-} - s^{p_t^+}$	0.6612	0.5324	0.2408	0.6157	0.6148	0.6152	0.3439	0.3275	0.3233	0.4519	0.4518	0.4523
Case IV												
$m^{p_t} - c^{p_t}$	0.5209	0.5087	0.4821	0.5837	0.5714	0.5639	0.3876	0.3924	0.3947	0.4738	0.4796	0.4846

$m^{p_t^+} - c^{p_t^+}$	0.5170	0.5053	0.4786	0.4835	0.4649	0.4530	0.2370	0.2473	0.2545	0.4481	0.4547	0.4596
$m^{p_t^-} - c^{p_t^-}$	0.5121	0.6668	0.7704	0.5214	0.5046	0.4934	0.2367	0.2668	0.2934	0.4243	0.4360	0.4470
Case V												
$m^{p_t^+} - c^{p_t}$	0.5226	0.5104	0.4859	0.6369	0.6433	0.6441	0.4162	0.4226	0.4278	0.4579	0.4647	0.4714
$m^{p_t^+} - c^{p_t^+}$	0.5187	0.5070	0.4824	0.5496	0.5546	0.5536	0.2726	0.2848	0.2952	0.4314	0.4391	0.4457
$m^{p_t^+} - c^{p_t^-}$	0.5138	0.6679	0.7721	0.5826	0.5876	0.5866	0.2723	0.3034	0.3319	0.4069	0.4199	0.4328
Case VI												
$m^{p_t^-} - c^{p_t}$	0.5543	0.3867	0.0069	0.6607	0.6670	0.6721	0.4396	0.4363	0.4430	0.4702	0.4765	0.4840
$m^{p_t^-} - c^{p_t^+}$	0.5507	0.3825	0.0002	0.5791	0.5842	0.5887	0.3018	0.3018	0.3139	0.4442	0.4514	0.4589
$m^{p_t^-} - c^{p_t^-}$	0.5462	0.5840	0.5598	0.6099	0.6150	0.6190	0.3015	0.3199	0.3497	0.4203	0.4327	0.4463

Source: Computed by the authors

Note: $a^{p_t} - b^{p_t}$ represent the C-T results when a^{p_t} is the unrestricted model and b^{p_t} is the restricted model. s^{p_t} , m^{p_t} and c^{p_t} indicate single-factor, multiple-factor and combined-factor models respectively, expressed as a function of aggregate oil price. Disaggregating oil price into positive and negative accordingly generates positive and negative superscript for p_t in s^{p_t} , m^{p_t} and c^{p_t} . Hence, $s^{p_t^+}$, $m^{p_t^+}$, $c^{p_t^+}$ and $s^{p_t^-}$, $m^{p_t^-}$, $c^{p_t^-}$ are positive and negative single-factor, multiple-factor and combined-factor models respectively, when positive and negative oil price are considered as predictor, respectively.

5.3.3 *Combined forecast vs Multiple-predictor forecast*

The RMSE presented for combined forecast in Table 6 is the simple average of the RMSE of the relevant single-predictor models. This is to examine whether multiple-factor model is also superior to the average of the forecast performance of oil price, inflation and output level. Our result from Table 6 reveals that multiple-factor model outperforms the combined factor model in predicting US stock prices. This is evident as no case of higher RMSE in multiple-factor model than combined factor model is recorded. In other words, the RMSE values in multiple-factor model are consistently lower than RMSE values for the combined forecast.

Since this result holds for both large and small cap stocks under both 50% and 75% in-sample data space and across the selected time horizons, it suggests that US stock is better predicted with multiple-factor model that combines inflation and output level with oil price. In other words, averaging the performance of single-factor models in the form of combined-factor forecast does not necessarily produce a substitute or superior forecast model for US stocks.

Meanwhile, the result of C-T test, which compares the unrestricted (multiple-case model) with the restricted (combined-factor case model), is also similar to what obtains comparing multiple case with single factor case. As revealed under the Cases IV

to VI in Table 7, predicting US stock with multiple-factor model is better than using a combined-factor model. This holds even when oil price is disaggregated into positive and negative in the combined-factor model. Furthermore, as noticed from in-sample and out-of-sample forecast for single predictor model, the result reveals that it is important to consider non-linearity in oil price when predicting large cap and small stocks in a single-factor model.

6. Policy implication of the findings

This study arrives at striking conclusions which have important policy implications. For instance, our predictability result reveals that oil price and its disaggregated negative and positive oil price changes, inflation and the level of real economic activities can predict both large cap and small cap US stocks significantly. However, this result is clarified by the in-sample analysis and supported by the out-of-sample analysis that US stocks (large and small caps) are better predicted with domestic factors (inflation and output level) than with external factor (oil price). This suggests that it will be irrational for investors or investment analysts to predict US stock prices without taking cognizance of US domestic macroeconomic fundamentals, and particularly inflation and output level. More so, since the result holds for both large and small caps, then investment analysts must consider the influence of US domestic macroeconomic fundamentals whether forecasting large cap or small cap stock.

In addition, the fact that inflation and economic activities predict US stock prices suggests that government agencies should always aim at minimizing the imbalances in the economy due to their implications on the stock market.

7. Summary and concluding remarks

This study argues that the single-factor (oil price-based) predictive model for US stock market proposed by NG (2014) may not be adequate to capture both external and internal economic imbalances that seem to have significant implications on the US stock market. Hence, it proposes a multiple-factor case, motivated by the APT framework, whereby oil price is combined with selected domestic macroeconomic indicators such as inflation and the level of real economic activities. Basically, it contributes to the extant literature on the predictability of US stock market in three ways. First, it examines whether oil price predicts US stocks better in the proposed multiple-factor predictive model. Secondly, it examines whether categorizing US stocks into large cap and stock cap matters in modeling and forecasting the US stock market. Thirdly, it employs recently developed Feasible Quasi Generalized Least Squares (FQGLS) estimator by WN (2014) to capture persistence, endogeneity and heteroscedasticity in the multiple-factor model. Furthermore, the study employs the rolling window approach and selects the better predictive model between the single-factor, combined forecast and multiple-factor models using Campbell-Thompson test. It also considers a wide range of time horizon (ranging from 6 to 18 months) and different in-sample data space (50% and 75% of the total observation) for robustness check.

Our result reveals that US stock market is better predicted in a multiple-factor model that comprises oil price, inflation and level of real economic activities. This result is explained by the rejection of the null hypothesis of no predictability for all the predictors in both simple and multiple predictive regressions and the fact that multiple-factor models outperform the single-factor variants. However, this result is clarified in the in-sample analysis and supported by the out-of-sample analysis that US stocks (large and small caps) are better predicted with domestic factors (inflation and output level) than with external factor (oil price). This implies that the inclusion of these domestic factors should not be jettisoned when forecasting the US stock market.

More importantly, the result shows that it is important to categorize US stocks into large cap and small cap when predicting US stock market whether in the single-factor or multiple-factor model. This occurs as the two categories of US stock market appear to respond differently to positive and negative oil price changes both in the in-sample and out-of-sample forecasts. For instance, negative oil price is a weak predictor of large cap stock but a good predictor of small cap stock in the single-factor and combined forecast scenarios (see Table 6). More so, the out-of-sample regression result suggests that while inflation is the best predictor for small cap stocks, the large cap stock is better predicted by output than oil price and inflation. Since the different categories of stock caps react differently to the same set of predictors, it then implies

that a single model cannot be used to generalize for the two categories of stocks, and doing so may lead to erroneous conclusions.

In summary, the multiple-factor predictive model outperforms the single-factor variants for both in-sample and out-of-sample forecasts. Also, generalizing the predictability of US stock market with large cap may lead to misleading inferences. In addition, it may be necessary to pre-test the predictors for persistence, endogeneity and conditional heteroscedasticity particularly when modeling with high frequency series. Our results are robust to different forecast measures and forecast horizons.

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Appendix

Dealing with endogeneity, persistence and conditional heteroscedasticity in the predictive model for stock returns

To test for endogeneity and persistence and also determine an estimable predictive regression model that accounts for these features, we follow the procedure as detailed in the LW (2004) and NB (2015). Let us assume the following predictive model for the stock price as previously specified:

$$r_t = \alpha + \beta x_{t-1} + \varepsilon_t; \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \quad (1A)$$

where r_t is as previously defined and x_t is a potential predictor variable of stock price which is restricted to oil price, output and inflation. We can also assume an AR(1) process for x_t (see also Stambaugh, 1986, 1999; Mankiw and Shapiro, 1986; Nelson and Kim, 1993; LW, 2004; NG, 2014) :

$$x_t = \phi + \rho x_{t-1} + \varepsilon_{x,t}; \quad \varepsilon_{x,t} \sim N(0, \sigma_{\varepsilon_{x,t}}^2) \quad (2A)$$

Both ε_t and $\varepsilon_{x,t}$ are expected to be correlated for endogeneity bias to pose any serious concern on the outcome of the forecast and $\hat{\rho} < 1$, an assumption that is required for stationarity. The null hypothesis of no predictability is given as $H_0 : \hat{\beta} = 0$. However, in the presence of persistence and endogeneity, LW (2004) shows that $\hat{\beta}$ exhibits a bias which can be expressed as $E(\hat{\beta} - \beta) = \gamma(\hat{\rho} - \rho)$. The direction of bias (i.e. upward or downward) is determined by γ and $\hat{\rho}$. If for instance γ is positive (based on the nature of relationship between ε_t and $\varepsilon_{x,t}$); then, a downward bias in $\hat{\rho}$ causes a downward bias in $\hat{\beta}$. Since x_t is expected to be stationary for any meaningful forecast to be conducted, then, the bias in $\hat{\beta}$ is at most $\gamma(\hat{\rho} - 1)$. In the absence of persistence, the bias disappears (although there is still potential correlation between the two errors due to endogeneity bias); however, if there is existence of persistence, then, $\hat{\rho} \neq 0$ and any predictive model that ignores the information in $\hat{\rho}$ tends to understate the predictive power of the predictor(s). As demonstrated in the literature, any predictive regression model that accounts for the information in $\hat{\rho}$ significantly strengthens its forecasting accuracy (see LW, 2004 for a review).

To test for the degree of persistence with the null hypothesis of $H_0 : \hat{\rho} = 1$, we estimate equation (2A) with OLS estimator and the first lag autoregressive coefficient is used to determine the extent of persistence. The closer is this coefficient to 1, the higher is the degree of persistence. Also, to test and capture the endogeneity effects, we follow the WN (2012, 2014)⁵ approach which establishes the relationship between the two errors (ε_t and $\varepsilon_{x,t}$) as follows:

$$\varepsilon_t = \gamma\varepsilon_{x,t} + \eta_t \quad (3A)$$

where ε_t and $\varepsilon_{x,t}$ are as previously defined, η_t has a zero mean and variance $\sigma_{\eta_t}^2$ and $E(\varepsilon_{x,t}\eta_t) = 0$. In other words, η_t is the remainder error of ε_t after controlling for $\varepsilon_{x,t}$ and therefore by construct, $\varepsilon_{x,t}$ and η_t are not expected to be correlated. The null hypothesis of no endogeneity bias is given as $H_0 : \gamma = 0$ which can be tested by replacing the error terms in equation (3A) by their corresponding residuals obtained from the estimation of equations (1A) and (2A). By way of substitution and re-arrangement, we can re-write equation (11) as:

$$r_t = \delta + \beta x_{t-1} + \gamma(x_t - \rho x_{t-1}) + \eta_t \quad (4A)$$

where $\delta = \alpha - \gamma\phi$. In order to correct for the inherent bias in β , LW (2004) suggests bias-adjusted OLS estimator of β which is described as:

$$\hat{\beta}_{adj} = \hat{\beta} - \gamma(\hat{\rho} - \rho) \quad (5A)$$

⁵The same approach was adopted by NG (2014) and NB (2015) in the forecasting of stock returns.

Since ρ is unknown, LW (2004) suggests we can put a lower bound on $(\hat{\rho} - \rho)$ by assuming that $\rho \approx 1$ (in line with the null hypothesis for testing for unit root) while $\hat{\rho}$ is determined from equation (2A). We follow the same approach in this paper to estimate the $\hat{\beta}_{adj}$. Note that in the absence of persistence and endogeneity effects, $\hat{\beta}_{adj} = \hat{\beta}$ since $\gamma(\hat{\rho} - \rho) = 0$ and by implication $\gamma(x_t - \rho x_{t-1}) = 0$. Therefore, in the absence of persistence and endogeneity effects, equation (4A) reduces to equation (2A) and the standard OLS is valid for estimation in that instance. Similarly, if there is no evidence of endogeneity but the predictor variable still exhibits persistence; so far the order of integration for the predictor does not exceed $I(1)$, the equation (1A) is a valid predictive regression model. However, if there is no evidence of persistence but endogeneity bias is evident; then equation (4A) becomes:

$$r_t = \delta + \beta x_{t-1} + \gamma x_t + \eta_t \quad (6A)$$

where $\hat{\rho} \approx 0$ and as a consequence, bias-adjusted OLS estimator of β in (5A) adjusts to: $\hat{\beta}_{adj} = \hat{\beta} + \gamma\rho$ and where $\rho \approx 1$, $\hat{\beta}_{adj} = \hat{\beta} + \gamma$. In this case, the adjustment in $\hat{\beta}_{adj}$ is determined by the sign on γ . For instance, a negative correlation between the two errors captured by γ implies that $\hat{\beta}$ adjusts upwards when $\hat{\rho} \approx 0$.

In addition to endogeneity and persistence effects, dealing with conditional heteroscedasticity becomes necessary for financial markets like stock markets that are susceptible to shocks and volatility. We follow the approach proposed by WN (2012, 2014) to account for conditional heteroscedasticity in the model. WN (2012) proposes a FQGLS estimator which exploits the information contained in the conditional heteroscedastic variance of the regression residuals in order to generate more precise estimates. Their estimator assumes the regression error, that is η_t , follows an autoregressive conditional heteroskedastic (ARCH) structure - $\hat{\sigma}_{\eta,t}^2 = \mu + \sum_{i=1}^q \varphi_i \hat{\eta}_{t-i}^2$, and the resulting $\hat{\sigma}_{\eta,t}^2$ can be used as a weight in the predictive model (see NG, 2014). The estimation of the weighted predictive model by OLS is described as the FQGLS estimator. Essentially, the GLS-based t-statistic for testing $\beta = 0$ is given as (see also NG, 2014):

$$t_{FQGLS} = \frac{\sum_{t=q_m+2}^T \tau_t^2 x_{t-1}^d r_t^d}{\sqrt{\sum_{t=q_m+2}^T \tau_t^2 (x_{t-1}^d)^2}} \quad (7A)$$

where $\tau_t = 1/\sigma_{\eta,t}$ is used in weighting all the data in the predictive model and

$$x_t^d = x_t - \sum_{s=2}^T x_s / T.$$

We subject equations (1) to (8) in the main text to this procedure and thereafter forecast with the appropriate model on the basis of the observed underlying features of the predictors.

To account for these features in the proposed augmented predictive models, we further extend the LW (2004), NG (2014) and NB (2015) single-predictor forecast model to account for a multiple-predictor model. In the case of two predictors for example, we express the model as follows:

$$r_t = \alpha + \beta_1 x_{1,t-1} + \beta_2 x_{2,t-1} + \varepsilon_t \quad (8A-A)$$

where x_1 and x_2 are two potential predictors of stock price. The error term - ε_t has a mean of zero and variance of σ_ε^2 . Like the previous analyses, we can assume a first order autoregressive model for x_1 and x_2 :

$$x_{1t} = \phi_1 + \rho_1 x_{1,t-1} + \varepsilon_{x_1,t} \quad (8A-B)$$

$$x_{2t} = \phi_2 + \rho_2 x_{2,t-1} + \varepsilon_{x_2,t} \quad (8A-C)$$

where $\varepsilon_{x_1,t}$ and $\varepsilon_{x_2,t}$ are assumed to have zero mean values and $\sigma_{\varepsilon_{x_1}}^2$ and $\sigma_{\varepsilon_{x_2}}^2$ as their respective variances. The null hypothesis of perfect persistence - $H_0 : \rho = 1$ is tested against the alternative of - $H_1 : \rho < 1$. To capture endogeneity effects, we assume the following correlation between ε_t and $\varepsilon_{x_1,t}$ and $\varepsilon_{x_2,t}$:

$$\varepsilon_t = \gamma_1 \varepsilon_{x_1,t} + \gamma_2 \varepsilon_{x_2,t} + \eta_t \quad (9A)$$

Since η_t is the remainder error of $\varepsilon_{r,t}$ after accounting for $\varepsilon_{x_1,t}$ and $\varepsilon_{x_2,t}$, it is therefore important to assume that $E(\varepsilon_{x_1,t}\eta_t) = E(\varepsilon_{x_2,t}\eta_t) = 0$. To implement equation (9A), it is also important to assume no correlation between $\varepsilon_{x_1,t}$ and $\varepsilon_{x_2,t}$ which can easily be verified by expressing one against the other [say $\varepsilon_{x_1,t} = \delta\varepsilon_{x_2,t} + v_t$] and thereafter test for the significance of the relevant parameter - δ . The null hypothesis of no endogeneity in equation (9A) - $H_0 : \gamma_1, \gamma_2 = 0$ is tested against the alternative of $H_1 : \gamma_1, \gamma_2 \neq 0$.

By substitution and further simplification of equation (9A), we can re-write equation (7A) as follows:

$$\pi_t = \omega + \beta_1 x_{1,t-1} + \beta_2 x_{2,t-1} + \gamma_1 (x_{1t} - \rho_1 x_{1,t-1}) + \gamma_2 (x_{2t} - \rho_2 x_{2,t-1}) + \eta_t \quad (10A)$$

where $\omega = \alpha - \phi_1 \gamma_1 - \phi_2 \gamma_2$ and the corresponding equation for the bias-corrected OLS estimates for the β 's is given as:

$$\pi_t = \omega + \beta_{1,adj} x_{1,t-1} + \beta_{2,adj} x_{2,t-1} + \gamma_1 (x_{1t} - \rho_1 x_{1,t-1}) + \gamma_2 (x_{2t} - \rho_2 x_{2,t-1}) + \eta_t \quad (11A)$$

where $\hat{\beta}_{1,adj} = \hat{\beta}_1 - \hat{\gamma}_1 (\hat{\rho}_1 - 1)$ and $\hat{\beta}_{2,adj} = \hat{\beta}_2 - \hat{\gamma}_2 (\hat{\rho}_2 - 1)$. It is easy to verify that equation (11A) reduces to equation (1A) in the absence of persistence and endogeneity effects.